

Analysis method 1 is used to determine the critical temperature of the infinite lattice by using the data for the order parameter of the lattice with  $L = 4$ . The lower limit for the temperature interval is taken as the value where the distortions on the curve(s) begin. This happens at  $T = 13.664$  for the lattice with  $L = 4$ .<sup>34</sup> The data for the order parameter within the interval  $13.664 \leq T \leq 14.370$  are fit to the power law with correction in Eq. (29), using the criterion  $0.49 \leq \beta \leq 0.51$ . In doing this,  $T_c$  is varied within the interval  $14.820 \leq T_c \leq 15.000$ . The results for the critical temperature and the critical amplitude of the infinite lattice are  $T_c = 14.898 \pm 0.010$  and  $B = 1.9 \pm 0.1$ , respectively. At this value of the critical temperature,  $\epsilon \geq 0.035$  for the upper limit of the temperature interval is satisfied, implying that the size-effects in these data are negligibly small.<sup>22,35</sup> The value of the critical temperature is in agreement with the  $1/d$ -expansion result of  $T_c = 14.892$ <sup>78</sup> and the recent value of  $T_c = 14.8911 \pm 0.0002$ .<sup>79</sup>

## 5. Conclusion

The review of the results obtained from the simulation of the Ising model on the Creutz cellular automaton leads to the following conclusions: it is a very good approximation to the Ising model above a limiting temperature. This temperature region includes a sufficiently large interval below the critical temperature of the magnetic susceptibility where the infinite lattice can be approximated. The degree of approximation is revealed by the simulations with increased accuracy. Presently for the critical temperatures, the third digit after the decimal point, and for the critical exponents, the second digit have been reached in accuracy, and within this accuracy the theoretical predictions and the Monte Carlo results are reproduced. This review shows that the Creutz cellular automaton can be used as an alternative research tool for Ising model investigations, and it is now at the stage of testing the predictions of the theory, rather than being tested.

## Acknowledgments

It is a pleasure to thank D. Stauffer for his encouragement for writing this review, and for his comments and suggestions and for providing the recent preprints throughout all the work on the subject.

The author thanks P. M. C. de Oliveira for drawing his attention to Ref. 5.

## References

1. N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller, *J. Chem. Phys.* **21**, 1087 (1953).
2. R. J. Glauber, *J. Math. Phys.* **4**, 294 (1963).
3. R. H. Swendsen and J. S. Wang, *Phys. Rev. Lett.* **58**, 86 (1987).
4. U. Wolff, *Phys. Rev. Lett.* **62**, 361 (1989).
5. K.-C. Lee, *J. Phys.* **A28**, 4835 (1995); C. M. Care, *J. Phys.* **A29**, L505 (1996).
6. M. Creutz, *Phys. Rev. Lett.* **50**, 1411 (1983).
7. R. C. Desai, D. W. Heermann, and K. Binder, *J. Stat. Phys.* **53**, 795 (1988); D. W. Heermann and R. C. Desai, *Comput. Phys. Comm.* **50**, 297 (1988).
8. G. Y. Vichniac, *Physica* **D10**, 96 (1984).
9. S. Wolfram, *Theory and Applications of Cellular Automata* (World Scientific, Singapore, 1986).
10. H. J. Herrmann, *J. Stat. Phys.* **45**, 145 (1986).
11. J. G. Zabolitzky and H. J. Herrmann, *J. Comput. Phys.* **76**, 426 (1988).
12. M. Schulte, W. Stiefelhagen, and E. S. Demme, *J. Phys.* **A20**, L1023 (1987).
13. S. C. Glotzer, D. Stauffer, and S. Sastry, *Physica* **A164**, 1 (1990).
14. W. M. Lang and D. Stauffer, *J. Phys.* **A20**, 5413 (1987).
15. U. M. S. Costa and H. J. Herrmann, *J. Stat. Phys.* **47**, 597 (1987).
16. D. Stauffer, *J. Phys.* **A24**, 909 (1991).
17. D. Stauffer, *Int. J. Mod. Phys.* **C8**, 1263 (1997).
18. D. Stauffer, APS Atlanta Meeting (1999).
19. M. Creutz, *Ann. Phys.* **167**, 62 (1986).
20. B. Kutlu and N. Aktekin, *J. Stat. Phys.* **75**, 757 (1994).
21. N. Aktekin, *Physica* **A232**, 397 (1996).
22. N. Aktekin, *Int. J. Mod. Phys.* **C**, to be published.
23. N. Aktekin, A. Günen, and Z. Sağlam, *Int. J. Mod. Phys.* **C**, to be published.
24. N. Aktekin and A. Günen, unpublished.
25. N. Aktekin, 3rd General Congress of BPU (Cluj-Napoca, Romania, 1997).
26. B. Kutlu and N. Aktekin, *Physica* **A208**, 423 (1994).
27. B. Kutlu and N. Aktekin, *Physica* **A215**, 370 (1995).
28. B. Kutlu, *Physica* **A234**, 807 (1997).
29. B. Kutlu, *Physica* **A243**, 199 (1997).
30. C. Dress, *J. Phys.* **A28**, 7051 (1995).
31. N. Aktekin, *Physica* **A219**, 436 (1995).
32. N. Aktekin, M. Sağlam, and B. Boyacıoğlu, unpublished.
33. N. Aktekin, Ş. Erkoç, and M. Kalay, manuscript in preparation.
34. N. Aktekin, *Int. J. Mod. Phys.* **C8**, 287 (1997).
35. O. G. Mouritsen and S. J. Knak Jensen, *Phys. Rev.* **B19**, 3663 (1979).
36. V. Privman, ed. *Finite-Size Scaling and Numerical Simulation of Statistical Systems* (World Scientific, Singapore, 1990).

37. J. Rudnick, H. Guo, and D. Jasnow, *J. Stat. Phys.* **41**, 353 (1985); D. Jasnow, "Finite-size scaling, hyperscaling and the renormalization group," in *Finite-Size Scaling and Numerical Simulation of Statistical Systems*, ed. V. Privman (World Scientific, Singapore, 1990) p. 99.
38. R. Kenna and C. B. Lang, *Phys. Lett.* **B264**, 396 (1991).
39. R. Kenna and C. B. Lang, *Nucl. Phys.* **B393**, 461 (1993).
40. K. Binder, M. Nauenberg, V. Privman, and A. P. Young, *Phys. Rev.* **B31**, 1498 (1985).
41. X. S. Chen and V. Dohm, *Physica* **A251**, 439 (1998); E. Brezin, *J. Phys.* **43**, 15 (Paris, 1982).
42. M. Suzuki, *Prog. Theor. Phys.* **58**, 1142 (1977); Y. Y. Goldschmidt, *Nucl. Phys.* **B285**, 519 (1987).
43. R. Kretschmer, K. Binder, and D. Stauffer, *J. Stat. Phys.* **15**, 267 (1976).
44. S. Tang and D. P. Landau, *Phys. Rev.* **B36**, 567 (1987).
45. C. Munkel, D. W. Heermann, J. Adler, M. Gofman, and D. Stauffer, *Physica* **A193**, 540 (1993).
46. M. Gofman, J. Adler, A. Aharony, A. B. Harris, and D. Stauffer, *J. Stat. Phys.* **71**, 1221 (1993).
47. D. Stauffer and R. Knecht, *Int. J. Mod. Phys.* **C7**, 893 (1996).
48. D. Stauffer and J. Adler, *Int. J. Mod. Phys.* **C8**, 263 (1997).
49. D. Stauffer, *Int. J. Mod. Phys.* **C10** (1999).
50. D. Stauffer, *Physica* **A244**, 344 (1997).
51. N. Ito, *Physica* **A196**, 591 (1993).
52. B. Damman and J. D. Reger, *Europhys. Lett.* **21**, 157 (1993).
53. K. B. Lauritsen and H. C. Fogedby, *J. Stat. Phys.* **72**, 189 (1993).
54. P. H. Poole and N. Jan, *J. Phys.* **A23**, L453 (1990).
55. K. MacIsaac and N. Jan, *J. Phys.* **A25**, 2139 (1992).
56. F. Wang and M. Suzuki, *Physica* **A223**, 34 (1996).
57. M. P. Nightingale and H. W. J. Blöte, *Phys. Rev. Lett.* **76**, 4548 (1996).
58. M. Siegert and D. Stauffer, *Physica* **A208**, 31 (1994).
59. Z. Salmon and J. Adler, *Int. J. Mod. Phys.* **C9**, 195 (1998).
60. H. W. J. Blöte, E. Luijten, and J. R. Heringa, *J. Phys.* **A28**, 6289 (1995).
61. A. M. Ferrenberg and D. P. Landau, *Phys. Rev.* **B44**, 5081 (1991).
62. A. L. Talapov and H. W. J. Blöte, *J. Phys.* **A29**, 5727 (1996).
63. C. F. Baillie, R. Gupta, K. A. Hawick, and G. S. Pawley, *Phys. Rev.* **B45**, 10438 (1992).
64. M. E. Fisher and J.-H. Chen, *J. Phys.* **46**, 1645 (Paris, 1985).
65. B. G. Nickel and J. J. Rehr, *J. Stat. Phys.* **61**, 1 (1990).
66. J. C. Le Guillou and J. Zinn-Justin, *J. Phys.* **48**, 19 (Paris, 1987).
67. S. Wansleben and D. P. Landau, *Phys. Rev.* **B43**, 6006 (1991).
68. R. Bausch, V. Dohm, H. K. Janssen, and R. K. P. Zia, *Phys. Rev. Lett.* **47**, 1837 (1981).
69. H. O. Heuer, *J. Phys.* **A25**, L567 (1992).

70. N. Ito, *Physica* **A192**, 604 (1993).
71. A. I. Larkin and D. E. Khmel'nitskii, *Zh. Eksp. Teor. Fiz.* **56**, 2087 (1969); *Sov. Phys. JETP* **29**, 1123 (1969); E. Brezin, J. C. Le Guillou and J. Zinn-Justin, *Phys. Rev.* **D8**, 2418 (1973).
72. D. S. Gaunt, M. F. Sykes, and S. McKenzie, *J. Phys.* **A12**, 871 (1979).
73. A. J. Guttmann, *J. Phys.* **A14**, 233 (1981).
74. K. K. Mon, *Europhys. Lett.* **34**, 399 (1996).
75. G. Parisi and J. J. Ruiz-Lorenzo, *Phys. Rev.* **B54**, R3698 (1996); G. Parisi and J. J. Ruiz-Lorenzo, *Phys. Rev.* **B55**, 6082 (1997).
76. H. W. J. Blöte and E. Luijten, *Europhys. Lett.* **38**, 565 (1997).
77. M. Cheon, I. Chang, and D. Stauffer, *Int. J. Mod. Phys.* **C10**, 131 (1999).
78. M. E. Fisher and D. S. Gaunt, *Phys. Rev.* **A133**, 224 (1964).
79. J. Berger, unpublished.