Preface

Mathematical knowledge and sophistication, computational power, and areas of application are expanding at an enormous rate. As a result, the demands on the training of applied mathematicians are increasing all the time. It is therefore not easy to decide what should constitute the core mathematical training of an applied mathematician. We take the view that every applied mathematician, whatever his or her ultimate area of interest may turn out to be, should have a grounding in the fundamentals of analysis.

The aim of this book is to supply an introduction for beginning graduate students to those parts of analysis that are most useful in applications. The material is selected for its use in applied problems, and is presented as clearly and simply as we are able, but without the sacrifice of mathematical rigor.

We focus on ideas of central importance, and attempt to avoid technicalities and detours into areas of more specialized interest. While we make every effort to motivate the ideas introduced, and include a variety of examples from different fields, this book is first and foremost about analysis.

We do not assume extensive mathematical prerequisites of the reader. The book is intended to be accessible to students from a wide variety of backgrounds, including undergraduate students entering applied mathematics from non-mathematical fields, and graduate students in the sciences and engineering who would like to learn analysis. A basic background in calculus, linear algebra, ordinary differential equations, and some familiarity with functions and sets should be sufficient. We occasionally use some elementary results from complex analysis, but we do not develop any methods from complex analysis in the text.

We provide detailed proofs for the main topics. We make no attempt to state results in maximum generality, but instead illustrate the main ideas in simple, concrete settings. We often return to the same ideas in different contexts, even if this leads to some repetition of previous definitions and results. We make extensive use of examples and exercises to illustrate the concepts introduced. The exercises are at various levels; some are elementary, although we have omitted many of the routine exercises that we assign while teaching the class, and some are harder and are an excuse to introduce new ideas or applications not covered in the main text. One area where we do not give a complete treatment is Lebesgue measure and integration. A full development of measure theory would take us too far afield, and, in any event, the Lebesgue integral is much easier to use than to construct.

In writing this book, the material has expanded beyond what can be covered in a year long course for beginning graduate students. When teaching a three-quarter course, we usually cover Chapters 1–5 in the first quarter, which provide a review of advanced calculus and discuss the basic properties of metric and normed spaces, followed by Chapters 6–9 in the second quarter, which focus on Hilbert spaces, including Fourier series and bounded linear operators. In the last quarter, we cover a selection of topics from Chapters 10–13, which discuss Green's functions, unbounded operators, distribution theory, the Fourier transform, measure theory, function spaces, and differential calculus in Banach spaces. The choice and emphasis of the topics depends on the backgrounds and interests of the students.

The material presented here is standard. Many of the sources we have drawn upon are listed in the bibliography. The bibliography is not comprehensive, however, and is limited to books that we feel will be useful to the intended audience of this text, either for background reading, or to pursue in greater depth some of the topics treated here.

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