

Contents

<i>Preface</i>	vii
1. Introduction and Background	1
1.1 Interaction of Physics and Mathematics	1
1.2 Manifolds: Smoothness and Other Structures	4
1.3 The Basic Questions	6
1.4 Some Basic Topological Exotica	9
1.4.1 Whitehead continua	9
1.4.2 Weierstraß functions	10
1.5 The Physics of Certain Mathematical Structures	10
1.6 The Physics of Exotic Smoothness	13
1.7 In Sum	13
2. Algebraic Tools for Topology	15
2.1 Introduction	15
2.2 Prerequisites	16
2.3 Concepts in Algebraic Topology	20
2.3.1 Homotopy groups	21
2.3.2 Singular homology	25
2.4 Interplay between Homotopy and Homology	31
2.5 Examples	32
2.6 Axiomatic Homology Theory	33
2.7 Conclusion	34
3. Smooth Manifolds, Geometry	35
3.1 Introduction	35

3.2	Smooth Manifolds	35
3.3	de Rham Cohomology	41
3.4	Geometry: A Physical/Historical Perspective	49
3.5	Geometry: Differential Forms	52
4.	Bundles, Geometry, Gauge Theory	55
4.1	Introduction	55
4.2	Bundles	55
4.3	Geometry and Bundles	62
4.3.1	Connections	64
4.4	Gauge Theory: Some Physics	69
4.5	Physical Generalizations, Yang-Mills, etc.	82
4.6	Yang-Mills Gauge Theory: Some Mathematics	83
5.	Gauge Theory and Moduli Space	85
5.1	Introduction	85
5.2	Classification of Vector and Principal Fiber Bundles	86
5.3	Characteristic Classes	101
5.4	Introduction of Spin and Spin_C Structures	114
5.5	More on Yang-Mills Theories	119
5.6	The Concept of a Moduli Space	124
5.7	Donaldson Theory	126
5.8	From Donaldson to Seiberg-Witten Theory	135
6.	A Guide to the Classification of Manifolds	151
6.1	Preliminaries: From Morse Theory to Surgery	153
6.1.1	Morse theory and handle bodies	153
6.1.2	Cobordism and Morse theory	159
6.1.3	Handle bodies and surgery	161
6.2	Application of Surgery to Low-dimensional Manifolds	169
6.2.1	1- and 2-manifolds: algebraic topology	169
6.2.2	3-manifolds: surgery along knots and Thurston's Geometrization Program	173
6.3	Higher-dimensional Manifolds	177
6.3.1	The simply-connected h-cobordism theorem	177
6.3.2	The non-simply-connected s-cobordism theorem*	180
6.4	Topological 4-manifolds: Casson Handles*	182
6.5	Smooth 4-manifolds: Kirby Calculus	187

6.6	Why is Dimension 4 so Special?	190
6.7	Constructing 4-manifolds from Intersection Forms	193
6.7.1	The intersection form	193
6.7.2	Classification of quadratic forms and 4-manifolds	197
6.7.3	Some simple manifold constructs	200
6.8	Freedman's Classification	203
7.	Early Exotic Manifolds	205
7.1	Introduction	205
7.2	Some Physical Background: Yang-Mills	206
7.3	Mathematical Background: Sphere Bundles	207
7.4	Milnor's Exotic Bundles	208
7.5	Coordinate Patch Presentation	211
7.6	Geometrical Consequences	213
7.7	Eells-Kuiper Smoothness Invariant	215
7.8	Higher-dimensional Exotic Manifolds(Spheres)	215
7.9	Classification of Manifold Structures	221
8.	The First Results in Dimension Four	231
8.1	The Smoothing of the Euclidean Space	231
8.2	Freedman's Work on the Topology of 4-manifolds	234
8.3	Applications of Donaldson Theory	237
8.4	The First Constructions of Exotic \mathbb{R}^4	239
8.4.1	The first exotic \mathbb{R}^4	241
8.5	The Infinite Proliferation of Exotic \mathbb{R}^4	243
8.5.1	The existence of two classes	245
8.6	Explicit Descriptions of Exotic \mathbb{R}^4 's	248
8.7	Other Non-compact 4-manifolds	250
9.	Seiberg-Witten Theory: The Modern Approach	253
9.1	The Construction of the Moduli Space	254
9.2	Seiberg-Witten Invariants	257
9.3	Gluing Formulas	259
9.4	Changing of Smooth Structures by Surgery along Knots and Links	260
9.5	The Failure of the Complete Smooth Classification	263
9.6	Beyond Seiberg-Witten: The Cohomotopy Approach	264
10.	Physical Implications	267

10.1	The Principle of Relativity	267
10.2	Extension of Metrics	270
10.3	Exotic Cosmology	272
10.4	Global Anomaly Cancellation of Witten	275
11.	From Differential Structures to Operator Algebras and Geometric Structures	281
11.1	Exotic Smooth Structures and General Relativity	281
11.2	Differential Structures: From Operator Algebras to Geo- metric Structures on 3-manifolds	291
11.2.1	Differential structures and operator algebras	292
11.2.2	From Akbulut corks to operator algebras	297
11.2.3	Algebraic K-theory and exotic smooth structures	301
11.2.4	Geometric structures on 3-manifolds and exotic differential structures	303
	<i>Bibliography</i>	307
	<i>Index</i>	317