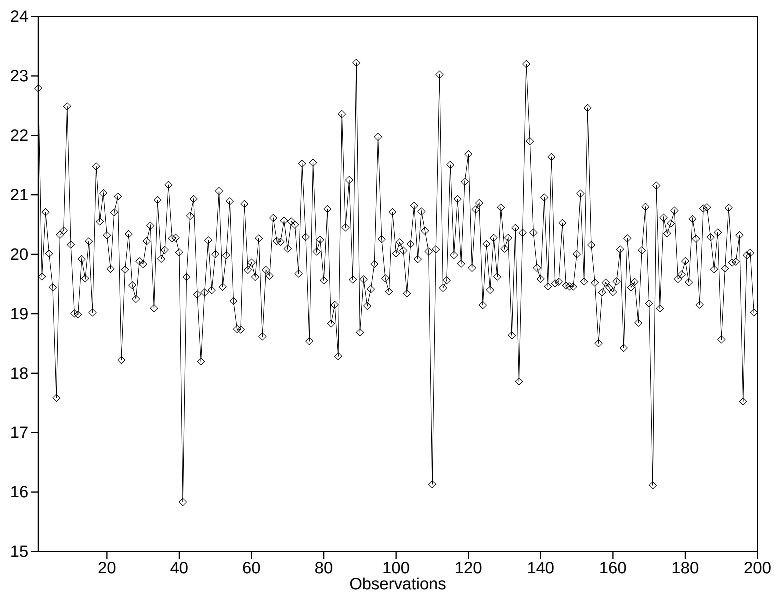
4. An Example

The training data set plotted in Fig. 2(a) is composed of $m = 40$ subgroups of $n = 5$ observations, corresponding to an "in control" process. We have estimated $\hat{m}_1 = 19.99$, $\hat{\mu}_2 = 1.11$, $\hat{\gamma}_1 = -0.27$, and $\hat{\gamma}_2 = 2.96$. If we use the methodology suggested in the appendix, we have $\hat{\delta_1} = -1.61$ and $\hat{\delta_2} = 8.64$. Because $-1.96 < \hat{\delta_1} < 1.96$ and $\hat{\delta_2} > 1.645$ with a confidence level $1 - \alpha = 0.95$, we can conclude that the data distribution is significantly symmetrical and leptokurtic. In Fig. 2(b) we plotted the standard deviation for the $m = 40$ subgroups. We notice that the UCL $= 1.89$ of the standard deviation chart, assuming normality for the data, leads three points "out of control". Thus, if the value UCL $= 1.89$ is used for the control of the production, we can expect numerous false "out of control" signals. Because the distribution of the data seems to be symmetrical and leptokurtic we suggest to use a symmetrical Johnson S_U transformation. We plotted in Fig. 2(c) the standard deviation of the data transformed using Eq. (5) with $\hat{b} = 1.617$ and $\hat{d} = 1.39$ (estimated using Eqs. (3) and (4)). Assuming that the transformed data are quasi-normal $N(0, 1)$, we immediately have (for $n = 5$) UCL $= 2.089$. We can notice now that there is no more point "out of control".

5. How Kurtosis Affects Classical Charts

The goal of this section is to investigate when the method proposed in this paper can favorably be used and when it cannot, and to give a very general rule concerning



(a)

Fig. 2. An example.

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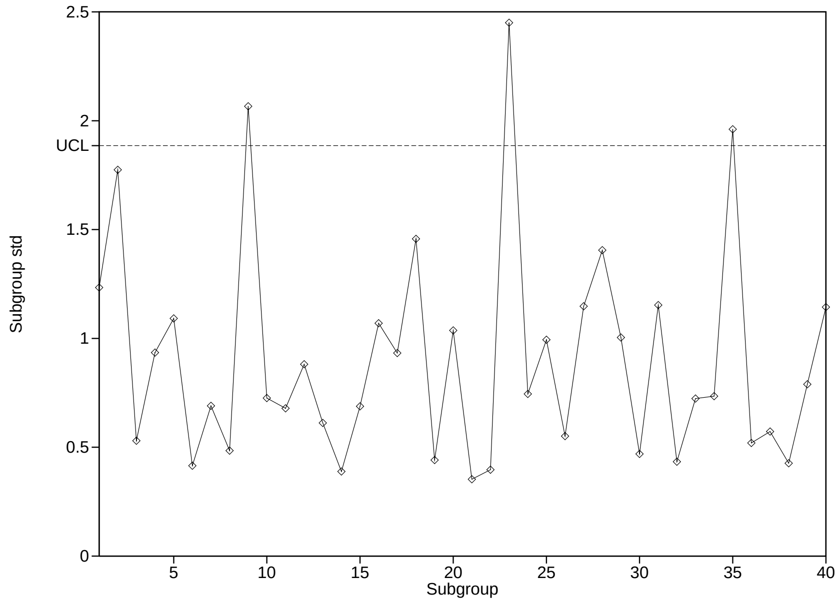
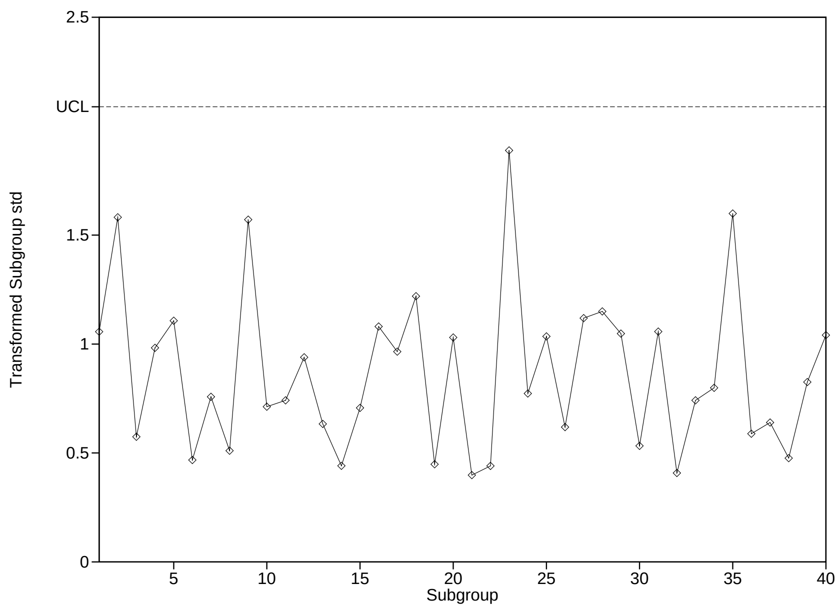


(b)



(c)

Fig. 2. (*Continued*).

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its use. The computation of the limits of the "classical" control charts assumes that the distribution of the data must be normally distributed. If this is not the case, the type I error α really obtained will be different than the expected type I error $\alpha = 0.0027$ (3σ limits). But how large is the difference between the observed and the expected type I error when the kurtosis increases? In order to evaluate the impact of the kurtosis γ_2 of the data on the observed type I error we used the following approach:

- for a sample size $n \in \{5, 7, 9\}$.
- for a kurtosis $\gamma_2 \in [0, 10]$, mean $m_1 = 0$, and variance $\mu_2 = 1$.
- compute b and d using Eqs. (3) and (4).
- generate (by inverse simulation of Eq. (2)) a set of m samples of n symmetrical Johnson S_U random variables having parameters (b, d). The number m of sample has been chosen such that the total number of generated data is $m \times n = 3\,465\,000$.
- compute the mean, median, standard deviation, and range for each sample.
- compute the proportion of data outside the control limits for each of the four statistics. For a kurtosis $\gamma_2 = 0$, the estimated observed proportion of data outside the control limits (the type I error) must be close to the expected one $\alpha = 0.0027$, for all the control charts.

In Fig. 3 we have plotted the observed type I error versus the kurtosis for sample size $n = 5, 7, 9$. The conclusions of these simulations are:

- The mean and median charts seem to be very insensitive to the kurtosis of the data. As expected, this is particulary true for the median. The larger the sample size, the more insensitive the charts.
- In contrary, the standard deviation and range charts seem to be very sensitive to the kurtosis. The range chart is the most sensitive. The larger the sample size, the more sensitive the charts.

From this, we can conclude that the method proposed in this paper is mostly devoted to the computation of the limits of standard deviation or range charts (dispersion charts in general) of data having a positive kurtosis, even if it can also be applied to the other statistics.

6. OC and ARL Curves

Let X be a symmetrical Johnson S_U with parameters (b, d). Without loss of generality, we will assume that $m_1 = 0$. By definition, the random variable Z defined by:

$$Z = b \sinh^{-1}\left(\frac{X}{d}\right) \tag{6}$$

is a normal $(0, 1)$ random variable. In order to compute the OC and ARL curves of a control chart using the method proposed in this paper, we have to first find the distributions of the random variables U and V defined as:

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