Preface

Time to gather stones...

In his well-known popular lectures R. Feynman [1] reflects on the way physical theories are built up and distinguishes two such ways or, rather, two stages in the process of their construction: *(i)* the "Babylonian" stage and *(ii)* the "Greek" stage.

It is not difficult to guess that the term "Babylonian" refers to ancient Babylon and the corresponding physical theory is just geometry. A Babylonian geometer (the words "mathematician" or "physicist" were not yet coined) knew many facts about circles, triangles, and other figures, and his understanding was not purely empirical because he could also *relate* different such facts with each other. He could e.g. heuristically derive a fact A on the basis of some other facts B and C or maybe the fact C on the basis of B and some other fact D, etc. He was actually quite efficient in his main job: the calculation of the areas of land estates. In other words, his theory described the observed experimental facts well and had direct practical applications.

Our Babylonian colleague was lacking, however, a consistent structured system in which a set of basic simple facts are chosen as *axioms* and all others are rigorously derived as *theorems*. Constructing such a system meant the advanced "Greek" stage of understanding geometry. At this stage a physical theory becomes a branch of mathematics where the primary criterion for Truth is not Experiment, but rather Rigor of derivations and internal logical Consistency. Feynman writes that a modern physicist is a

Babylonian rather than a Greek in this respect: he does not care too much about Rigor, and his God and ultimate Judge is Experiment.

Strictly speaking, this is not quite correct (a general statement never is). Some branches of classical and also of quantum physics have now quite reached the Greek stage. As a result, many problems concerning classical and quantum dynamics of Hamiltonian systems are studied now in mathematics departments rather than in physics departments. If you want, rendering a theory understandable to mathematicians may be considered a final goal of research in physics.

Regarding quantum chromodynamics and quantum field theory in general, we are living now in interesting times when we go over from the Babylonian to the Greek stage. A basic concept in quantum field theory is the path integral. Mathematicians still do not understand what it is: they cannot provide a rigorous definition for it. My personal impression, however, is that such a mathematical definition will be given soon.

A transitional period when a physical theory is already essentially constructed and understood at a semi-heuristic level but has not become yet a branch of mathematics, is a proper time to write textbooks. And many such textbooks were written recently. Is there really a need to write another one? It is difficult for me to judge whether this particular book is worth reading, but if you ask me why I have written it, the answer is the following:

There are very good recent monographs [2; 3] on quantum field theory. They are modern and comprehensive but are rather voluminous, and this might be considered a disadvantage. The large size of these books is due to two major choices which their authors make:

- The basics of quantum field theory are described from scratch. No preliminary acquaintance with the subject is required.
- *Many* different quantum field theories are considered there: quantum electrodynamics, quantum chromodynamics, and electroweak theory. The third volume of Weinberg's book is devoted to supersymmetric theories.

Besides general comprehensive monographs, there are also books (cf. [4]–[7]) written in a different style and specially devoted to Yang–Mills theory and to QCD. All these books are "recommended reading" and many pages in my own book were actually written under their influence. That especially concerns Faddeev's and Slavnov's book which was one of my own primary sources when learning QCD.

All these books concentrate, however, on the *perturbative* aspects of QCD, whereas my primary goal is to convey to the reader what is currently known about *nonperturbative* QCD dynamics. In particular, I discuss in some details the instanton solution, the nonperturbative (*viz.* lattice) definition of the path integral including also the "Ginsparg–Wilson revolution" — the modern way to define fermionic path integrals. I further discuss the θ vacuum picture, theoretical and phenomenological aspects of quark–hadron duality, the QCD phase transitions brought about by heat or by squeeze, and, last but not least, confinement. To make the book reasonably self-contained, the perturbative dynamics of QCD is also considered, however. We outline the derivation of the Feynman rules, dwell upon the ultraviolet regularization and renormalization issues, and also review the structure of infrared and collinear singularities in QCD.

Only quantum chromodynamics is discussed * and we assume the reader to be already familiar with QED, the book [8] being our main reference point. That means the acquaintance with the standard operator approach to quantization, derivation of the Feynman rules using Wick's theorem, calculation of Feynman integrals, etc. We describe, however, path integral quantization in field theory and in quantum mechanics — a material not covered by the book [8], which can, of course, be found in almost *any* modern textbook on field theory.

We put emphasis on the principal foundational aspects of QCD and provide only a limited discussion of QCD phenomenology. Actually, one of our assumptions is that the reader has a rough idea about hadron phenomenology, knows about flavor $SU(3)$ symmetry (still, the most basic facts about unitary groups are given in the Appendix) and its implications for the spectrum of mesons and baryons, has heard about confinement, etc.

All these limitations together with a concise manner of presentation make the book relatively thin (which was one of my goals). Being influenced by the style of the Landau course, I have put in a number of **Problems** accompanied by **Solutions**. Each such **Problem** examines a particular (usually, technical and, more often than not, relatively simple) theoretical issue.

*To be more exact, we discuss sometimes QED and also some hypothetical (not describing the real World) field theories, but we do it only to clarify and to illustrate what happens in QCD. Purely theoretical issues having no direct implications for QCD are not discussed.

The book does not contain many references. I have decided *not* to give references to classical original papers whose results are now well known and are discussed also in other books on QCD. Vanity-ridden, I sometimes do not refer to my own papers. I tried to quote, however, the *names* of the people who obtained the results I write about. Also, I give references to some less known papers, or to papers which are known well but are more recent and did not get into the textbooks yet. In other words, the references are quite chaotic and always (with one exception) have a nature of personal recommendation to *read* this or that particular book or a paper.

This book has grown up from the lecture course on the foundations of QCD which I have given for graduate students in high energy theory in ITEP in Moscow during the Spring of 1998, and which I later put in writing. Of course, when actually writing the book, I have added much more material than I was able to squeeze into the real time lectures and I also added some "off-shell" lectures which were never actually given. Thus, the book which you hold now in your hands differs from the original draft in an essential way. Still, I have decided to preserve the form of the virtual lecture course.

It is a pleasure to thank C. Bender, M. Chernodub, J. Hoppe, M. Lavelle, P. Mansfield, M. Peskin, K. Selivanov, M. Shifman, A. Slavnov, A. Vainshtein, and L. Yaffe for discussions and useful remarks concerning the book. Special thanks are due to M. Staudacher, who took pains to read carefully the whole manuscript, for many valuable remarks and comments and to my wife E. Savvina for her very essential help with editing the manuscript. I appreciate also the artistic contribution of Olga Feigelman who has drawn the headpieces for the parts.