Preface

The purpose of this textbook is to create a general framework for the study of optimal iterative procedures for problems that are solved approximately. For generality the setting is abstract, but we present many applications to practical problems allowing readers to take advantage of the most modern high-speed calculating devices and provide examples to illustrate concepts and major theorems. The examples are selected from Astrophysics (Radiative Transfer, Kinetic Theory of gasses), Mechanics (elasticity), Economics (Predator-Prey problems), the n-dimensional Euclidean space and other applied areas.

This textbook is an outgrowth of research work undertaken by us during the past ten years. Such a comprehensive study of optimal iterative procedures appears to be needed and should benefit not only those working in the field but also those interested in, or in need of, information about specific results or techniques. At the same time we also hope to provide here a textbook for senior undergraduate and graduate numerical analysis courses in this area, and in order to meet this aim, we have endeavored to make the main text as self contained as possible, to provide all results in full detail, and to include a number of exercises throughout the textbook. In order to make the study useful as a reference source, we have complemented each section with a set of "Remarks" in which literature citations are given, other related results are discussed, and various possible extensions of the results of the text are indicated. For completion, the book ends with a contemporary list of algorithms used throughout this text (see Appendix) and a comprehensive list of references.

Because we believe our readers come from diverse backgrounds and have varied interests, (such as Numerical Analysis, Numerical Functional Analysis, Approximation Theory, Differential-Integral equations, Mathematical Economics, Engineering, Physics, Applied Mathematics and Applied Science) we provide "recommended reading" throughout the textbook.

Often a long textbook summarizes knowledge in a field. This textbook, however, may be viewed as a report on work in progress. We provide a foundation for a scientific field that is rapidly changing. Therefore we list numerous conjectures and open problems as well as alternative models which need to be explored. There are many more questions which we have chosen not to list.

Some readers would like to pick up specific information from an appropriate section of a chapter without having to refer back to earlier sections in order to comprehend concepts appearing in the section of interest. That is why we have tried to make the sections as independent of each other as possible. That is why the reader will find material that could have been implied by information presented in earlier sections.

The main textbook (Chapters 3–8) presupposes that the reader is familiar with the material covered in multivariate calculus and linear algebra courses. Some of this material is reviewed and collected in the form we need in Chapters 1 and 2.

In particular, Chapter 1 treats divided differences of nonlinear operators, Fréchet-derivatives and the connection between them. In Chapter 2, constants and functions appearing frequently in error analysis of iterative procedures are recognized. Convergence and optimal error estimates are given in Chapters 3–8 for different numerical methods such as Newton's method, Modified Newton's method, Secant method, Stirling's method, Steffenson's method, Midpoint method, Jarratt method and other variations of the so-called Newton-like methods. In particular in Chapters 3–6 we use hypotheses (Lipschitz-type) on the First Fréchet-derivative, whereas in Chapters 7 and 8 hypotheses on the second Fréchet-derivative are used.