$$i\hbar \frac{\partial A}{\partial t} = -i\hbar\ c \frac{\partial A}{\partial x} . \tag{1.19}$$

The $i\hbar$ cancels so no $\hbar$ appears in the wave equation. A more familiar form comes from writing $H^2 = c^2p^2$, which gives

$$\frac{\partial^2 A}{\partial t^2} - c^2 \frac{\partial^2 A}{\partial x^2} = 0. \tag{1.20}$$

This is exactly the result of writing one of Maxwell's Equations , $\nabla\times\mathbf{H} = -(1/c)\ \partial\mathbf{E}/\partial t$ (if there are no currents present), combined with Eqs. (1.17). $4\pi c j_z$ would appear in place of the zero on the right in Eq. (1.20) if there were currents present, with j_z the current density in the z-direction.

The treatment of light and of electrons is entirely parallel, as it must be in accordance with the wave-particle duality. For classical physics it is the wave description of light which is familiar and the particle picture of the electron, but both descriptions are appropriate in both cases. Different kinds of systems have different dynamical relations between momentum and energy and correspondingly different wave equations. Given the equations for light, we can introduce a refractive index which varies with position, and therefore a $c(\mathbf{r})$ and study the dynamics of the photons, or refraction of the light.

1.4. New Meaning for Potentials

The vector potential **A** which we introduced in Section 1.3 is an invention, just as the classical electrostatic potential ϕ , is an invention. In classical physics the vector potential only has meaning through the defining equations, Eqs. (1.17), which relate the observable fields to it. If we add a constant to the vector potential (or to the electrostatic potential) it does not change the fields and we regard it as a simple definition, like the definition of an origin to a coordinate system, x, y, z . It is playing for the photon the role played by the wavefunction ψ which we also regarded as an invention. It may not be surprising that in quantum mechanics these electrostatic and vector potentials take on real new meaning.

This meaning is associated with the Aharanov-Bohm Effect (Aharanov and Bohm (1959)). They proposed two experiments, which appeared to be paradoxes. One is for an electron wave, illustrated in Fig. 1.2. We imagine an electron packet moving from the left, and then being split into two packets, which finally recombine on the right and produce a diffraction pattern on a luminescent screen to the right, just as light - or light wave packets - will form a diffraction pattern with constructive and destructive interference on a film in a two-slit experiment. Now we add two Faraday cages, conducting cages shown by dashed lines, and while the packets are entirely within the cages we apply a potential difference between them. No electric field is seen by either packet; their relative potential is simply shifted. However, according to the Schroedinger Equation, Eq. (1.16), this constant shift will advance the phase of one wavepacket [$i\hbar\,\partial\psi/\partial t = H\psi$ means that the rate of change of the phase is equal to the energy divided by $\hbar$ as we shall see more completely in Eq. 1.22).] relative to the other. If we keep it on long enough to shift the relative phase by π, and then again put the potential difference to zero, the points on the screen at which constructive and destructive interference occurs will be interchanged. The potential is removed before the packets reach the cage surfaces but the interference pattern is modified even though no field has ever been felt by the packets. In this sense the potential, or potential difference, has taken on new physical meaning with measurable consequences. The electrostatic potential which was invented to describe electric fields obtains new meaning in quantum mechanics. It is natural at first to try to dismiss the paradox by saying that there will be small leakage fields within the cages, but that is not the point. Effects such as that can be made as small as one likes and the physical consequences of the phase shift remain large.

There is an important message from this exercise. Once one is sure that the argument is correctly made, there is no real need to test it experimentally, any more than one needs to test a proposed perpetual motion machine once one sees that it violates the first law of thermodynamics. One does better to adjust one's intuition. Our feeling that only the electric field has consequences is generalized here to an electron which in some sense is in two places, and then the potential difference between the two places has consequences. Considerable experimental effort goes into displaying the

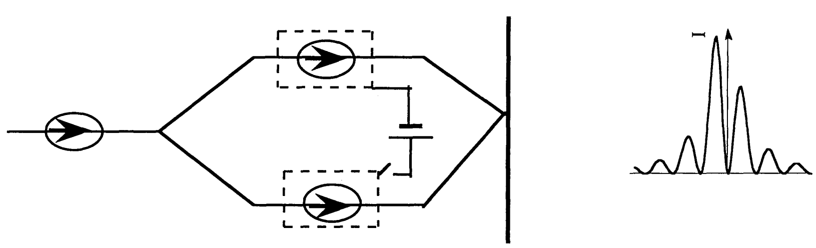

Fig. 1-2. The Aharanov-Bohm Paradox. An electron packet from the left is split into two packets, which pass through two Faraday cages. A potential difference applied to the cages, while the packets are in them, will advance the phase of one packet relative to the other and shift the diffraction pattern, though no field ever exists where the wavefunction is nonzero.

surprising consequences of quantum theory, but it is clear that one is not "testing" quantum mechanics, but testing one's understanding of quantum mechanics. We can think of the starting postulate as an absolute truth, certainly on the scale of the truth of conservation of energy.

The second Aharanov-Bohm Paradox split an electron packet so that the packets went on opposite sides of a long coil containing a magnetic field. The field in such a long coil is contained entirely within the coil so that neither packet ever feels a magnetic field. However, the vector potential is nonzero outside the coil and its presence will also shift the fringes. This "fictitious" vector potential has physical consequences. Similarly our fictitious wavefunctions will have important physical consequences which we explore in this text.

1.5 Measurement

The discussions above have touched on the question of measurement, which receives considerable attention from physicists. Our more pragmatic view is based upon Eq. (1.1) which tells us how the average of many measurements (in this case of position x) is predicted using $\psi(x)^*\psi(x)$. Quantum mechanics can tell us what sets of circumstances are consistent with each other. It tells us what we will see on the screen in the experiment shown in Fig. 1-2. From a practical point of view, that is what is needed. We should not speculate "which path the electron followed". We could set up another experiment which would also detect which way it went, and we would predict, and find, that the interference pattern would disappear, as we shall see in detail in Section 23.4.

People have sought ways to avoid the problem, as in the consideration of fringing fields discussed above for the Aharanov-Bohm experiment, by thinking of many electrons interfering with each other. However, that experiment can be done with so few electrons passing through per second that there is almost no chance of two electrons being in the apparatus at once, and the same result is obtained. It is again certainly better to adjust our intuition to fit the truth, rather than the other way around.

We may make a classical analogy as to how quantum mechanics tells us what is consistent, though it may or may not be helpful to follow such an analogy. Imagine walking past the window of a pool hall and noting a tall and a short man playing pool. The tall man is about to hit the cue ball aimed at the five ball. You estimate that it is an easy shot and the five ball should go in the corner pocket. From where the cue ball will then be he will probably choose to put the three ball in the side pocket. After you pass the window you recognize that maybe the five ball will *not* go in and an entirely