surprising consequences of quantum theory, but it is clear that one is not "testing" quantum mechanics, but testing one's understanding of quantum mechanics. We can think of the starting postulate as an absolute truth, certainly on the scale of the truth of conservation of energy.

The second Aharanov-Bohm Paradox split an electron packet so that the packets went on opposite sides of a long coil containing a magnetic field. The field in such a long coil is contained entirely within the coil so that neither packet ever feels a magnetic field. However, the vector potential is nonzero outside the coil and its presence will also shift the fringes. This "fictitious" vector potential has physical consequences. Similarly our fictitious wavefunctions will have important physical consequences which we explore in this text.

1.5 Measurement

The discussions above have touched on the question of measurement, which receives considerable attention from physicists. Our more pragmatic view is based upon Eq. (1.1) which tells us how the average of many measurements (in this case of position x) is predicted using $\psi(x)^*\psi(x)$. Quantum mechanics can tell us what sets of circumstances are consistent with each other. It tells us what we will see on the screen in the experiment shown in Fig. 1-2. From a practical point of view, that is what is needed. We should not speculate "which path the electron followed". We could set up another experiment which would also detect which way it went, and we would predict, and find, that the interference pattern would disappear, as we shall see in detail in Section 23.4.

People have sought ways to avoid the problem, as in the consideration of fringing fields discussed above for the Aharanov-Bohm experiment, by thinking of many electrons interfering with each other. However, that experiment can be done with so few electrons passing through per second that there is almost no chance of two electrons being in the apparatus at once, and the same result is obtained. It is again certainly better to adjust our intuition to fit the truth, rather than the other way around.

We may make a classical analogy as to how quantum mechanics tells us what is consistent, though it may or may not be helpful to follow such an analogy. Imagine walking past the window of a pool hall and noting a tall and a short man playing pool. The tall man is about to hit the cue ball aimed at the five ball. You estimate that it is an easy shot and the five ball should go in the corner pocket. From where the cue ball will then be he will probably choose to put the three ball in the side pocket. After you pass the window you recognize that maybe the five ball will *not* go in and an entirely new scenario arises. The short man will pick up his cue and will probably seek to put the six ball in the other side pocket, etc. You could carry the first scenario, and the second scenario, as far as you like (with decreasing certainty of the details) and what you are really doing is determining sets of circumstances which are consistent with each other, but inconsistent with the circumstances of another scenario.

If you turn around and pass the window again, you may note that the five ball is still on the table: *scenario one has become irrelevant.* Indeed the short man is standing at the table, shooting at the six ball.

The scene is obviously chosen to indicate that the well-known "collapse of the wavefunction", which is supposed to occur when someone makes a measurement, is not a quantum phenomenon, but one of everyday classical experience. Schroedinger's cat should not to be of concern. More importantly, this analogy sets the stage: all we can do in quantum mechanics is to estimate sets of consistent circumstances, or scenarios, and the likelihood of each occurring. When we do an experiment, we eliminate - or make irrelevant - a large number of other scenarios. No other theory may ever do more than that.

1.6 Eigenstates

We have found that the observables position, momentum, and energy, are represented by operators on the wavefunction, and that a statistical average of measurements of such an observables O for a given wavefunction is given by $<O> = \int\psi^*O\psi\, d^3r / \int\psi^*\psi\, d^3r$ as in Eqs. (1.1) and Eqs. (1.13) through (1.15), but now written for wave functions in three-dimensions, $\psi(\mathbf{r},t)$. The d^3r indicates a volume integral. We are thinking of electrons, but this applies to the energy-density for light waves which can produce diffraction patterns such as we observed for electrons on the screen in Fig. 1.2. The mathematical consequences of this statement about statistical averages are extraordinary, and we turn to them next.

We do not focus on the mathematical details until we need them, but must mention that we always assume some set of boundary conditions on all wavefunctions, such as the condition that the wavefunction be nonzero only inside some surface, and therefore zero on the surface. It is also necessary that the operators be *Hermitian*, which means that for any two wavefunctions they satisfy $\int\psi_1^*(\mathbf{r})O\psi_2(\mathbf{r})d^3r = \{\int\psi_2^*(\mathbf{r})O\psi_1(\mathbf{r})d^3r\}^*$, which will always be true here and is readily verified for the operators we have introduced, using the boundary condition such as we just gave (partial integrations are required to prove it for the momentum and energy operators).