

Preface

The purpose of the present book is to explain, from a mathematical viewpoint, the theory of permanent progressive water-waves. If you look at waves on the surface of lake, you note that, as a rough approximation, the shapes of waves are invariant with respect to time, in a suitable coordinate system moving with a constant speed. These “permanent” waves are what we are going to study mathematically. Naturally, our task is to prove the existence of such waves, to study their geometrical properties, and to investigate their stabilities. The present book is concerned with the existence and the detailed properties of the solutions but the stability is mentioned only briefly. This lack of balance is due to the fact that we need too much mathematical preparation for stability.

The characteristic feature of water-wave motions is the fact that they are solutions to free boundary problems, in which differential equations must be solved in unknown domains. This causes nontrivial mathematical difficulties; if one wishes to avoid those difficulties, then one must resort to some simplified model in which differential equations are defined in a fixed, known domain. This is often called an approximate theory. Although such approximate theories are best suited to many research investigations of water-waves, particularly to the study of time-dependent motions, what is called the exact theory can be established if we restrict ourselves to permanent water-waves. Based on this view, we deal with water-waves as free boundary problems.

The water-waves are, in the present book, formulated as a bifurcation problem, as was done by Stokes, Levi-Civita and others. Among many important bifurcation parameters, the most important ones are the non-

dimensional gravity and the nondimensional surface tension. We will study the properties of water-waves by varying those parameters. The present book consists of two ingredients; numerical experiments and normal form analysis of the bifurcation equations. Many but not all the solutions can be explained by degenerate bifurcation equations, which are derived by means of the idea of the Golubitsky-Schaeffer theory.

Prerequisites for reading this book are an elementary knowledge of the Euler equations of motion for an incompressible inviscid fluid and some experience of elementary bifurcation theory. The readers are also expected to know functional analysis of elementary level. Although the numerical methods used in this book are well-known, we describe them as clearly as possible. Thus, we believe that a reader with elementary knowledge of numerical computation would have no difficulty in re-calculating our results with minimal labor.

Many of the contents of the present book are results of the authors: materials due to other researchers are selected by our taste, and graphs are re-computed by our own methods. Bias is therefore unavoidable, though we have tried to eliminate it. In order to offer the reader a wider view, we have collected as many references as we can, though an exhaustive list can not be hoped for. These references must be biased by our preference and there are surely many important research papers which we have missed. In addition, our misunderstanding, misprints, and inaccuracy in numerical computations are unavoidable, too. We would like to receive any comments on the present book, so we can improve it.

Finally we express our deepest gratitude to Professor A. D. D. Craik, whose comments improved this book substantially in both mathematical content and the way of presentation.