NON-STATIONARITY AS AN EMBEDDING PROBLEM

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Non-stationarity is the time dependent change in system dynamics. In this paper we describe a generalisation of nonlinear modelling techniques that may allow one to extract time dependent features from a nonstationary time series. Nonlinear modelling is generally applied to predict future values of a time series from past values. By explicitly considering the time as a variable in the model one may extend this methodology to encompass time dependent dynamics. This method has been applied to successfully describe: (i) a Shil'nikov mechanism from experimental string vibration data, (ii) a period doubling bifurcation in infant respiration, (iii) time dependent features in a computational simulation of cardiac arrhythmia and (iv) computational simulations of classical nonlinear systems.

1 Introduction

Radial basis models are capable of accurately and compactly modelling a wide variety of functional forms [1]. Judd and Mees [2] have shown that applying a radial basis modelling technique with a pseudo-linear optimisation and minimum description length [3] as a fitting criterion can be employed to accurately model short, noisy experimental data. This technique has been successfully applied to model (for example): annual sunspot numbers [2,4], human infant respiration [5,6,7,8], vowel intonations [4], ventricular arrhythmia [9,10] and classical nonlinear systems [2,11].

This nonlinear modelling technique proceeds by *embedding* a scalar time series in a high dimensional Euclidean space and building a map from the embedded space to the next scalar point. A simple extension of this methodology allows one to also embed a time component in the same phase space [12]. An extension of radial basis modelling to locally embedded basis functions (cylindrical basis modelling) [4] then allows one to efficiently build a model which has appropriate time dependent dynamics if and only if this is required. In this paper we describe the basic methodology, review some previous results and present some new applications.

Section 2 describes the modelling procedures we employ. Section 3 gives several examples of applications of this methodology and Section 4 provides some concluding remarks.

2 Modelling

Nonlinear modelling is logically and operationally divided into three important issues: (i) reconstruction, (ii) functional approximation, and (iii) model selection. In Section 2.1 we describe the reconstruction methodology. Section 2.2 describes the basic functional form of radial basis models and Section 2.3 describes the application of minimum description length for model selection. In Section 2.4 we describe the extension of radial basis modelling to cylindrical basis functions. Finally, Section 2.5 describes the operation of time embedding that we employ to model nonstationary data.

2.1 Reconstruction

The much cited Takens' embedding theorem [13] provides a guarantee that under certain conditions it is possible to reconstruct a finite dimensional dynamical system $(\Phi, \mathcal{X})$ (where $\Phi : \mathcal{X} \longmapsto \mathcal{X}$, $x_t \in \mathcal{X}$, $\dim(\mathcal{X}) = d$) from a single scalar time series $\{u_t\}_{t=1}^{N}$ where $u_t = g(x_t)$. In practice one cannot be sure that the observation function $g : \mathcal{X} \longmapsto \mathbf{R}$ is diffeomorphic (i.e. u_t may only observe a subsystem) and that the constraints on noise level and observation length N may not be met. However, a time delay embedding still allows one to reconstruct a noisy attractor with dynamics approximately equivalent to the underlying dynamical system (or a subsystem).

The time delay embedding is defined by

$$v_t = (u_t, u_{t-\tau}, u_{t-2\tau}, \ldots, u_{t-(d_e-1)\tau})^T$$

where τ and d_e are the *embedding lag* and *embedding dimension*. Embedding dimension and embedding lag may be selected by one of many criteria including: false nearest neighbours [14], zero of autocorrelation [15], minimum of mutual information [16] or one-quarter the quasi-period [8].

The *evolution operator* of the underlying dynamical system Φ ($x_{t+1} = \Phi(x_t)$ or the analogous for a continuous system) may then be approximated by a function F,

$$v_{t+1} = F(v_t) + e_t$$