Chapter 1

Foreign exchange markets

1.1 Introduction

They say that money never sleeps. The truth of this statement is apparent to anyone who has even passing familiarity with the inner workings of the world financial markets. Perhaps, the best confirmation can be found in the foreign exchange (forex) markets because of their depth, versatility and transparency. The sheer size of forex markets is mind boggling: the daily turnover is about $1.5 trillion. Changes in foreign exchange rates (FXRs) (i.e., relative prices of different currencies) are caused by both deep structural shifts in the respective economies and a variety of less fundamental factors. These changes have a profound impact on the world economy at large. An adequate formalism for studying the dynamics of FXRs has been developed by a number of researchers over the last thirty years. In addition to explaining the qualitative behavior of FXRs, this formalism can be used in order to develop consistent pricing of various derivative instruments, such as forwards, calls, puts, etc., whose value depends on the value of the underlying FXRs. By necessity this formalism is probabilistic in nature and requires a solid grasp of several mathematical disciplines for its efficient usage. In the present book we use these to solve a number of fundamental problems of financial engineering, such as derivative pricing, asset management, etc.

This Chapter is organized as follows. In Section 2 we give the a brief overview of the historic development of financial engineering. In Section 3 we discuss properties of forex as an asset class. In Section 4 we describe properties of spot FXRs. In Section 5 we introduce and discuss derivatives in the forex

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context. In Section 6 we give relevant references to the literature.

1.2 Historical background

In different disguises, problems of financial engineering have been discussed since classical times in a variety of sources stretching from Plato's Dialogues to the Old and New Testaments. Many illustrious noble houses in Medieval Italy made their fortunes by astute dealings in money lending and foreign exchange. An intimate relation between money lending (fixed income in modern language) and money changing (foreign exchange) was used by Italian bankers collectively known as the Lombards, in order to circumvent the church prohibition on charging interest on loans. They lent money in one currency and received it back in another one with the FXR artificially lowered to accommodate the interest. The blossoming of the Netherlands in the seventeenth century is, at least partly, due the introduction of new financial instruments such as forward contracts, calls and puts, etc. The rise of the British Empire resulted in further advances in finance, particularly, on the fixed income side. The economic growth in the United States was facilitated by the unprecedented growth of the financial system, especially, by the introduction of limited liability companies.

The modern development of financial engineering starts with the work of the French mathematician Louis Bachelier who, in the year 1900, published the now famous memoir entitled "*Theorie de la spéculation*". Bachelier's achievements are remarkable in several respects. To mention just one, he developed (before Einstein and others) the first theory of Brownian motion which he used in order to quantify the evolution of stock prices. In modern terms, Bachelier assumed that the stock price follows an arithmetic Brownian motion and, consequently, is distributed normally at any given time. He derived the pricing formulas for call and put options on such stocks. However, since Bachelier's theory predicted that stock prices can become negative and because of the sheer complexity of its mathematical apparatus, the theory was neglected by the mainstream economists for more than fifty years.

Fundamental contributions to modern financial engineering were made in the 1950s by several authors. Arrow (1953) and Debreu (1959) extended the existing economic models by incorporating uncertainty and showed how to solve the corresponding asset allocation problem. Modigliani and Miller (1958) proved that the financial structure of the firm, i.e., the firm's choice between equity and debt financing, does not affect its value. The method of financial arbitrage they used turned out to be even more useful than the theorem itself and became the method of choice for generations of financial engineers. Finally,

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