so attractive that additional risks are worth taking.

For simplicity we consider only investments in domestic and foreign fixed income instruments. In order to quantify uncertainties associated with these investments, we need to know the prices of domestic and foreign zero coupon bonds, i.e., the price in dollars (euros) at time $t = 0$ of the obligation to pay one dollar (euro) at time T in the future. We denote the price of the domestic and foreign bonds maturing at time T by $B^0_{0,T}$ and $B^1_{0,T}$, respectively. Assuming that the bond prices are deterministic (this is a strong assumption which is made in order to simplify the exposition), we can represent the bond prices at time t as

$$B^0_{t,T} = \frac{B^0_{0,T}}{B^0_{0,t}}, \qquad B^1_{t,T} = \frac{B^1_{0,T}}{B^1_{0,t}}.$$

In addition, we need to know the forex rate S_t, i.e., the number of dollars one needs to pay at time t in exchange for one euro. The dimension of S_t is dollar/euro. In contrast to bond prices which are deterministic in our simplified framework, the forex rate is random, so that its value at some future time T is uncertain. We use the domestic bond as a benchmark for measuring the rate of return on different investments. It is clear that the relative rate of return on investment in domestic bonds is zero. The relative rate of return on investment in foreign bonds is

$$\hat{r} = \frac{B^0_{0,T} S_T}{B^1_{0,T} S_0} - 1. \tag{1.1}$$

It can be both positive and negative. Thus, in order to achieve relative rates of return above zero, domestic investors have to put some of their money overseas.

For simplicity, in this chapter we assume that

$$B^0_{t,T} = e^{-r^0(T-t)}, \qquad B^1_{t,T} = e^{-r^1(T-t)},$$

where r^0 and r^1 are constant domestic and foreign interest rates, respectively.

1.4 Spot forex

Perhaps the simplest transaction in the forex market is the exchange of two currencies at the (fluctuating) spot rate prevailing at the time of the exchange. As a rule, all exchanges are conducted through middlemen called market makers, rather than directly between two interested parties, which explains the

so-called bid-ask spread, i.e., the difference between the lower rate at which the foreign currency can be sold to market makers and the higher rate at which it can be bought from them. (This concept will be familiar to anyone who has exchanged currency abroad.) Depending on the particular currency pair, the rate is agreed upon today while the actual transfer takes place either in two business days (US dollar/yen, euro/US dollar, pound/US dollar) or in one business day (US dollar/Canadian dollar). Instantaneous FXRs (as well as stock and bond prices) are determined by market forces (through the "invisible hand" envisioned by Adam Smith) and reflect their intrinsic values, as well as considerations of supply and demand. FXRs constantly fluctuate around their moving equilibrium values. Their most important characteristics are rates of return on investments in foreign currency (yearly, monthly, daily, hourly, etc.). Rates of return are random and have to be treated via statistical methods for studying time series. To give the reader an idea of what can be expected, we just mention that the distribution of daily returns for the foreign exchange rate USD/DEM over a period of ten years from 1986 to 1996 has volatility 0.11, skewness -0.1, kurtosis 5, and no daily deviations exceeded five standard deviations, while for the S&P 500 Index over the same period the distribution of returns has volatility 0.16, skewness -5, kurtosis 111, and that one (five) daily deviation (deviations) exceeded ten (five) standard deviations. The distribution of daily returns for the FXR is reasonably close to Gaussian, while for the S&P 500 Index the corresponding distribution is strongly non-Gaussian. In most cases it is not necessary to explain the observed FXRs and rates of return on investments in foreign currencies in fundamental terms; the main objective is to develop a model for pricing derivative instruments in terms of the underlying ones and to solve the asset management problem. Even though, in general, the distribution of the daily returns for underlying instruments is non-Gaussian, it is frequently assumed to be Gaussian for practical purposes. Surprisingly, more often than not, this approach produces satisfactory results.

Typical behavior of FXRs is illustrated in Figure 1.1.

1.5 Derivatives: forwards, futures, calls, puts, and all that

Exchange of currencies at spot rates serves only the most obvious and the most immediate needs of market participants. Their more sophisticated needs are met by derivative instruments. The basic types of forex and equity derivatives are forward and futures contracts, and calls and puts; their fixed income counterparts are known as forward rate agreements, Euros, caps, and floors.