

A major source of difficulty is the role of dissipation. Dissipation arises from the wall layer of the experimental flume and due to naturally occurring processes of surface wave-breaking. Many of laboratory and field observations are inseparably influenced by such dissipative mechanisms. Analysis of experimental measurements (Melville, 1982, 1983; Tulin and Waseda, 1999) indicate that energy transfer between wave components in the incipient stage of (and during) breaking happens rather rapidly in a complex way. Further understanding of this process is necessary to be able to enter the next stage of modelling in which post-breaking modulation may be calculated.

9. Summary

Wave transformation in the field involves wide-ranging processes including interactions among components and with varying depth and current. Modulation equations present a simpler framework with which both analytical and numerical insight and quantification of the processes may be achieved. An essential assumption in the derivation of the modulation equations is that length scale of variation of the amplitude envelope is much longer than that of the carrier waves. Thus, validity of such equations become limited when the wave properties (amplitude envelope, wave number) exhibit fast variation. With an ambient current, the current is assumed to be of large-scale compared to the waves. We also like to draw attention to the fact that 1D coherent wave groups (soliton-type solutions) are likely to become unstable in two dimensions. This is due to the instability of groups to oblique perturbations (e.g., Ablowitz and Segur, 1979). Because of this, coherent wave groups are more readily observed in wave flumes than in the field.

This review begins with some illustrative sections to guide uninitiated readers to the nonlinear processes in deep and intermediate water. Modulation equations involving cubic interactions have been presented for deep water followed by the more general cases of varying depth with or without an ambient current. Many theoretical descriptions deal with the classical case of deep water modulation. In engineering practices, the role of a finite and varying depth is important. The simplest form of equations on finite depth is the set of the so-called Davey–Stewartson equations [Eqs. (48)]. The cubic evolution equation on finite depth differs from the deep-water version in two ways. First, the coefficients of the equation depend on the depth. More importantly, the depth-averaged wave-induced current (long-wave) manifests itself in the amplitude modulation equation resulting in a coupling between the two. In 2D

propagation on water of finite depth, the evolution equation for the complex amplitude and the wave equation for the long-wave potential need to be solved simultaneously. In 1D, this is simplified since the long-wave potential can be obtained after the evaluation of the amplitude [Eqs. (60)].

Conservation laws and the special solutions of the NLS equation may be used to serve as good guidelines for the validation of the numerical models. With waves propagating over a varying depth, an associated process is the generation of free long waves. Long waves are of importance in regards to harbour oscillation and coastal sediment transport. Nonlinear modulation is not an essential mechanism behind the magnitude and generation of free long waves but adds to the features. Generation of long waves, driven by the wind waves, is discussed in some length in the section “generation of free long waves”. Effect of surface tension, although touched briefly in this review, is considered outside the present scope.

The motivations behind modulation equations of higher-order are more than only the natural theoretical extension. Both quantitative and qualitative changes are introduced through higher-order modulation equations. These changes agree with experimental measurements in a superior way than the cubic Schrödinger equations. For example, the side-band growth rate predicted by NLS equation is in satisfactory agreement with experimental measurement and exact computations only over a low range of initial steepness $(ka)_0 < 0.1$. This range is extended significantly by using the higher-order basis of Dysthe’s equation [Eqs. (115)]. Further, asymmetric growth rates of the upper and lower side bands could be at least qualitatively reproduced by this set. This observed feature of asymmetric growth rates is beyond the scope of the NLS equation. Dysthe’s equation (1979) is the first among the widely-known higher-order extensions of the NLS-type equation. Both Dysthe’s equation and the current-modified form given by Stocker and Peregrine (1999) are formulated for deep water only. Derived from the Hamiltonian principle, the Zakharov equation provides a broader basis for higher-order modulation equation. The equation proposed by Dysthe (1979) has been shown to be a special case of the Zakharov’s equation under the assumption of narrowness of the spectrum. Use of the Zakharov equation or a reduced form of it (where the reduction is based on the assumption of narrow-band approximation only in regards to nonlinearity) is recommended while considering a broad-band spectrum [Trulsen *et al.*, 2000 and Eqs. (168)]. Inclusion of an ambient current, allowing perhaps a depth-varying profile, is a desired future development for the higher-order formulations.

Experimental measurements have on one hand lent justification to the basis of nonlinear modulation equation. On the other hand, they have identified several gaps which motivate further developments. One area where theoretical developments are lacking is the modelling of dissipation. Apart from a few isolated attempts (e.g., Lake *et al.*, 1977; Trulsen and Dysthe, 1990), formulations of dissipation due to breaking and surface turbulence are largely absent in currently used modulation equations. This is in spite of the fact that dissipation is necessary in being able to model multi-cycle modulational evolution in many practical computations. It is also a generally held notion that dissipation is an essential mechanism behind frequency down-shift observed in uni-directional propagation. Notwithstanding the possibility that nonlinear interaction alone may cause a permanent down-shift in three-dimensional wave trains (Trulsen and Dysthe, 1997), breaking related dissipation remains an important aspect in future development of understanding the behaviour of evolution of steep waves of practical interest. Experiments by Melville (1982) and Tulin and Waseda (1999) indicate that the down-shift may be a rather sudden process associated with breaking instead of being gradual. Clearly, there is a strong need for a better understanding of such process both from experimental and theoretical investigations.

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