NONLINEAR MODULATION OF WATER WAVES

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1. Introduction

Customarily, in water-wave propagation problems, distinction is made between regular and random waves. Regular waves typically consist of a few components (not necessarily harmonics of each other) while random waves consist of many components in which the phases are distributed randomly, usually uniformly. Transformation of waves take place as they propagate due to interactions between components, variation of the bottom and current and forcing conditions such as wind.

In wave motion, effect of shear is usually confined to a thin boundary layer. Experiences show that the important features of nonlinear wave interactions, refraction and shoaling due to bottom and current variation may be represented satisfactorily using a simpler mathematical description under the assumption of the fluid being inviscid and irrotational. This allows introduction of a wave potential $\Phi(\boldsymbol{x}, z, t)$ such that the velocity field $(\boldsymbol{u}, w)^T$ is given by $(\nabla\Phi, \partial\Phi/\partial z)^T$ where $\boldsymbol{u} = (u, v)^T = (u_1, u_2)^T$ is the horizontal velocity vector and w is the vertical component. We shall use in this text a coordinate system such that $\boldsymbol{x} = (x_1, x_2)^T = (x, y)^T$ is directed horizontally while z is directed upwards (opposite to the gravity acceleration vector). The still-water level and the sea bed are defined respectively by $z = 0$ and $z = -h(\boldsymbol{x})$. The governing equations for wave motion are then given by a field equation, the Laplace equation for Φ, a kinematic and a dynamic condition at the free surface $z = \zeta(\boldsymbol{x}, t)$ and a kinematic condition at the bottom,

$$\nabla^2\Phi + \frac{\partial^2\Phi}{\partial z^2} = 0\,; \quad -h(\boldsymbol{x}, t) \le z \le \zeta(\boldsymbol{x}, t)\,, \tag{1a}$$