

Foreword

Cellular automata (CA) are among the simplest mathematical representations of complex systems; where, for the moment, we may take “complex system” to mean any dynamical system that consists of more than a few—typically nonlinearly-interacting parts. As such, CA are extremely useful idealizations of the dynamical behavior of many real systems, including physical fluids, neural networks, molecular dynamical systems, natural ecologies, military command and control networks, and the economy, among many others. Because of their underlying simplicity, CA are also powerful conceptual engines with which to study general pattern formation. They have already provided critical insights into the self-organization of chemical reaction-diffusion systems, crystal growth, pattern formation on seashells, and phase-transition-like phenomena in vehicular traffic flow, to name but a few examples. On a more practical side, CA may provide the basis for extremely powerful encryption algorithms, a subject about which there has recently been much heated debate. There is even some serious speculation that CA may provide the backbone of a radically new discrete fundamental physics (an idea that we will focus our attention on in Chapter 12).

While the history of CA can be traced back to early *Systems’ Theory* and rigorous mathematical analyses conducted primarily by Russian researchers in the 1930s and 40s, their more recent incarnation as simple models of complexity in nature can arguably be traced to a single landmark review paper published by Wolfram in the *Reviews of Modern Physics* in 1983 [wolf83a].

Indeed, the sudden, almost explosive, emergence of CA as a major scientific and intellectual discipline is one of the most important developments in the decade from 1975-1985. It is always difficult and, in fact, a bit arbitrary to pinpoint particular dates as the definitive starting point of a new development, but it is striking how after a slow beginning, much influenced by the scientific demands of World War II, ideas leading to CA and computers entered the literature at an increasing rate. Around 1970, about 300 to 400 mathematically oriented papers dealing with automata were published; at the same time, the number of physics papers was at most a tenth of that. Some ten years later (~ 1980), evidence of a

shift in interest was already appearing: the numbers of mathematics and physics oriented papers was now roughly comparable, 50 to 100 each. In the succeeding five years the physics, or physically motivated, papers increased dramatically (400 to 500), while the number of mathematics papers slowly declined. The number of physics papers continues to increase even today. It is clear that CA can be anticipated to play an increasingly significant, if not major, role in future physics, and these developments should be analyzed and evaluated.

In spite of the large number of papers published on CA, their direct impact on the *mainstream* of physics was (and, depending on what researcher you ask today, still is), rather minor. Although many research studies make use of the computer experience and expertise which has been (and continues to be) developed in connection with CA, this influence remains primarily technical and computational rather than conceptual or philosophical. The contributions of CA have so far consisted of somewhat disjoint investigations which are largely isolated from modern physics—although they are arguably an integral component of an emerging new interdisciplinary research genre, that includes physics, called *complexity theory* (about which we will have more to say later). Complexity theory—as defined by, say, research being done at the *Santa Fe Institute* in New Mexico—is very much an immature, even infant science. This fact, along with it being a fundamentally interdisciplinary field, precludes us from branding it as mainstream *physics*. However, as this book hopes to show, CA are being increasingly recognized as important both for natural ability to simulate the dynamics of real physical systems *and* for providing powerful conceptual models with which to build (possibly radically new) physical theories.

Whenever a new field emerges, many different individuals contribute to its development. This is of course also true for CA, yet three persons stand out as originating and shaping the field: Alan Turing (1936), John von Neumann (1948) and Stephen Wolfram (1983). In addition, Stanislaw Ulam (1950) and John Conway (1969) each made specific, original contributions of long-lasting fundamental significance. It is interesting that four of these founders were mathematicians, while the sole physicist worked in particle physics and cosmology (rather than many-body theory or macroscopic physics, where current CA techniques find their primary use).

The scientific, mathematical orientation of these founding fathers had a major impact on the early development of CA. It is most remarkable that von Neumann, arguably the dominant figure, came to CA via the unlikely path of an interest in formal logic and the foundations of mathematics. In the 1920s, many of the usual procedures in classical mathematics were severely criticized by Brouwer and Weyl, who argued that the methodology and philosophy of set theory were fundamentally unsound. The intuitionist dogma was that all mathematical results should be constructive: proofs and derivations should be obtained via finite algorithms. Hilbert suggested a systematic program to counter this intuitionist criticism.

John von Neumann, under Hilbert's influence, actually produced a paper in which he showed that a subsystem of classical analysis could be obtained in a fini-

tistic manner. He further showed, by finitistic means, that this system is free of contradictions. He hoped, and even conjectured, that this proof and general conclusion could be extended to all analysis. Kurt Godel's well known proof of the incompleteness theorem, completed a few years later (1931), showed that von Neumann's conjecture was wrong and that Hilbert's program to show the contradiction-free character of mathematics by intuitionist methods was hopeless.

An important consequence of von Neumann's familiarity with finite mathematical schemes was its close, if not direct, connection with numerical analysis. von Neumann, unlike the Bourbaki school of mathematics, had a deep personal interest in numerical results, as well as in qualitative results outside of pure mathematics. He strongly believed that numerical results could provide novel insights into many phenomena; insights which were not obtainable by exclusively analytical or abstract means. (He even believed that mathematical innovation itself had to originate—at least in part—from extra-mathematical sources: economic, biological, neurological, etc.) It was the confluence of these disparate factors, a knowledge of finite mathematical algorithms, and a deep interest in—and respect for—numerical results, that led to von Neumann's epoch making investigations of computers and automata.

It is perhaps a fortuitous but interesting coincidence that while in Princeton, von Neumann became well acquainted with Turing, who wrote his thesis on the "The Decision Problem," another one of Hilbert's problems in formal logic. von Neumann was clearly impressed by Turing: he asked Turing to be his assistant for the academic year 1938-1939. Although Turing did not accept the position, von Neumann was familiar with Turing's work; his general approach played an important role in von Neumann's later efforts to construct computers which could reproduce themselves. Thus, the succeeding explosive development of electronic computers is the curious progeny of a profound analysis of the logical foundations of mathematics on the one hand, and a highly personal interest in numerical results on the other. As unlikely (and presumably as incompatible) a lineage as one could possibly imagine! Yet these two factors were fundamental in starting the computer revolution and the emergence of CA. von Neumann used to say, "The computer changed intuitionistic mathematics from ideology to reality."

Although precursors of CA were studied in a rigorously mathematical context by Russian mathematician in the late 1920s and 1930s, CA, as such, were first introduced in discussions between Ulam and von Neumann in the fall of 1951. von Neumann was interested in finding a reductionist model for biological evolution. His ambitious scheme was to abstract a set of primitive local interactions necessary for the evolution of complex forms of organization essential for life. It was Ulam's felicitous suggestion to emphasize discrete systems and dynamics, which—since that time—is automatically included as a characterizing feature of CA and which initiated the explosive growth of the field.

During the succeeding summer of 1952, Fermi, Pasta and Ulam showed that the use of a computer, even when applied to a presumably well understood physical

situation in classical mechanics, can lead to unanticipated phenomena; phenomena that are not always easy to explain, even though the basic rules are simple. Using the rapidly increasing power of computers, many models could now be analyzed and discussed. Perhaps the best known model is Conway's *Life* game, a CA which, while based on almost absurdly simple rules, can exhibit extraordinarily complex and even bizarre behavior. Since the *Life* game can be played using rather modest computing facilities, it became very popular for a time—acquiring an almost cult-like following among computer “hackers” in the early 1970s. There was even a journal devoted exclusively to the publication of computer results (and their interpretations) of the *Life* game.^{||} Although this game did a great deal to publicize CA, the effect on the physics, mathematics and general scientific community was rather mixed. Whether causally related or not, the number of purely mathematical papers on CA dropped sharply after the publication of Conway's *Life* game in 1970.

For physicists, applied mathematicians, biologists, etc., the general popularity of the *Life* games raises an important question: *What do these games, or more generally, the processes generated by CA (in spite of their suggestive names), actually have to do with phenomena—physical or otherwise?* For actual applications and for the ultimate importance of CA as an autonomous scientific method, this is clearly a very basic question. Although a general answer to this question cannot yet be given, there are many indications that it is reasonable to hope that a thorough, deep understanding of CA may well lead to an alternative, possibly more profound, and more useful, formulation of the basic laws of nature. As a very minor hint, it has been observed by many physicists that CA are intimately related to discrete statistical models. Since these models are constructed to elucidate basic ideas and general principles of statistical mechanics, it is reasonable to expect that the corresponding CA possess a similar physical interpretation, and have similar physical relevance.

It is, of course, far from obvious that all or even most of the fundamental physics can be phrased in terms of discrete CA. In fact, in current particle and high energy physics, symmetries, gauge symmetries, groups, Lie groups, and differential geometry—all of which stress *continuity* and not discrete features—play increasingly important roles. Yet, even in fundamental physics, attempts are made (in fact, with surprising regularity!) to discretize space and time and to express the basic laws as difference equations. Even though none of these efforts have been spectacularly successful, it is still too early to categorically dismiss such attempts as being misdirected and necessarily pointless.

It is worth pointing out, that in one of the more successful areas of particle physics, *Quantum Chromodynamics* (QCD), where basic physical principles are

^{||}*Lifeline*: A Quarterly Newsletter for Enthusiasts of John Conway's game of *Life*, edited by Robert Wainwright, #'s 1-11, 1971-1973; available on the internet at URL address <http://members.aol.com/lifeline/life/lifepage.htm>. See also *Time* magazine, January 21, 1974: “The game of *Life*,” which is a lively account of hackers versus serious computer time.

presumably all known, the resulting equations are of such complexity that they can be explored only by using extensive computer simulations. In fact, the actual analysis is almost always performed in terms of a lattice gauge theory. The system is put on a lattice and treated numerically. This is not exactly a reduction to a problem in CA dynamics, but it is beginning to approach it. Certainly, many interpretation questions in lattice gauge theory contain elements CA theory. It is a highly nontrivial problem, for example, to decide whether solutions to the *discrete* problems—obtained numerically—are indeed legitimate (albeit approximate) solutions of the original *continuous* problems. The precise relation between a continuous problem and an approximate treatment of a related discrete problem is quite delicate and subtle. This, in spite of the claim made some time ago that the only problem remaining in strong interaction physics is to “get enough money for a supercomputer.” The precise arguments needed to assess the meaning of the results obtained by a numerical analysis of the discretized version of the theory, lead to discussions which involve many formal aspects of CA.

These two examples are just illustrations of the ubiquitous role of CA in many different, unrelated areas of physics. Through the important work of Wolfram [wolf83a], which strongly ignited the recent explosive development as far as physics is concerned, it has become clear that many highly complex phenomena are the result of the collective, cooperative dynamics of a very large number of—typically very simple—individual parts. In these emerging complex patterns, recognizable macroscopic features may appear that have no simple, visible, or immediate relation to the original microscopic rules. The mathematical analysis of these patterns and their underlying dynamics involves algebra, number theory, graph theory, logic, topology, as well as techniques originating from dynamical systems theory. (It is interesting, and a little ironic, that many of the questions arising from the study of such systems are closely related to the kind of problems von Neumann and Turing considered in their own studies of computability and the foundations of mathematics). Much of the current serious work with CA centers around the question of how simple rules produce complex patterns. It is certainly eminently reasonable to expect that these investigations will lead to important results. They already have. This alone would justify a serious systematic discussion.

But there still could be doubts. *Are CA merely an amusing form of computer graphics, a sophisticated computer game for scientists, or do they indeed herald a new beginning for a conceptually new and distinct approach to science generally and physics in particular?*

This book is intended to mitigate these doubts. There is already enough of a structure to the theory of CA to show that they provide an effective and practical basis for the treatment of specific, as well as general, questions. In this monograph, the physical, formal and mathematical framework will be systematized to such an extent, that the framework becomes the natural setting for an effective description of the natural world. Just to what extent the fundamental laws of physics can, or

should, all be phrased in terms of CA is impossible to say at this juncture. The subject is interesting enough, promising enough, and important enough, however, that a scientific field (or an individual scientist) ignores it at his or her own risk.