Preface

This monograph introduces the theory of stability and of time-optimal control of hereditary systems. It is well known that the future state of realistic models in the natural sciences, economics and engineering depends not only on the present but on the past state and the derivative of the past state. Such models that contain past information are called hereditary systems. There are simple examples from biology (predator-prey, Lotka–Volterra, spread of epidemics, e.g., AIDS epidemic), from economics (dynamics of capital growth of global economy), and from engineering (mechanical and aerospace: aircraft stabilization and automatic steering using minimum fuel and effort, control of a high-speed closed air circuit wind tunnel; computer and electrical engineering: fluctuations of current in linear and nonlinear circuits, flip-flop circuits, lossless transmission line). Such examples are used to study the stability and the time-optimal control of hereditary systems. Within this theory the problem of minimum energy and effort control of systems are also explored. Conditions for stability for both linear and nonlinear systems are given. Questions of controllability are posed and answered. Their application to the growth of the global economy yield broad, obvious but startling policy prescription. For the problem of finding some control to use in order to reach, in minimum time, an equilibrium point from any initial point and any initial state, existence theorems are given. Some effort is made to construct an optimal control strategy for the simple linear systems cited in the examples. For finite dimensional linear systems, time-optimal, and minimum effort feedback controls are constructed.

It is assumed that the reader is familiar with advanced calculus and has some knowledge of ordinary differential equations. Some familiarity with real and functional analysis is helpful. The contents of this book were offered as a year-long course at North Carolina State University in the 1989–1991 sessions. The students were mainly from engineering, economics, mathematical biology, and Applied Mathematics. Though in many aspects the book is self-contained, theorems often are stated and the appropriate references cited. The references should help a further exploration of the subject.

There are thirteen chapters in this book. The first chapter is devoted to examples from applications in which delays are important in time-optimal, minimum effort, and stability problems. The second chapter studies linear systems. The fundamental matrices of linear delay and linear neutral matrices are calculated and used to obtain the variation of parameter formulae for nonhomogeneous systems. The stability theory of linear and perturbed linear systems is considered.

Chapter 3 presents the basic Lyapunov and Razumikhin theories and their extensions by Hale and LaSalle and recently by Haddock and Terjéki. On the basis of these, a simple quadratic form is used to deduce a necessary and sufficient condition for stability of linear delay equations. The fourth chapter treats the global stability of functional differential equals of neutral type. The second part of the book begins in Chapter 5. It deals with the construction of minimum time and minimum effort optimal feedback control of linear ordinary differential systems. It is both an introduction and a model for such studies of linear hereditary systems. Most of the theory presented here is only available in research articles and theses. It will be most interesting to present parallel results for hereditary systems. Chapter 6 presents the theory for the underlying controllability assumptions required for the existence of optimal control of delay equations. The geometric theory of time-optimal control is then presented in Chapter 7, where numerous examples are considered. Here also the maximum principle both in Euclidean and function space is formulated. The fundamental work of Angell and Kirsch is stated: It gives a maximum principle in a setting that guarantees the existence of non-trivial regular multipliers for nonlinear problems involving point-wise control constraints. It presents an introduction to the synthesis of optimal feedback control of linear hereditary systems. The synthesis of optimal control of simple examples is attempted. In Chapter 8 controllability of nonlinear systems is treated. Interconnected systems are studied in Chapter 9. Their universal principles for the control of interconnected organizations are formulated. They have fundamental economic policy implications of immense practical interest. Chapters 10 through 12 consider the corresponding controllability and time-optimal control of linear and nonlinear neutral equations. Chapter 13 presents the stability theory of large-scale hereditary systems in the spirit of Michel and Miller.

The World

"Should we not question the very economic models often adopted by states which also as a result of international pressures and forms of conditioning cause the aggravate the situations of injustice and violence in which the life of whole people is degraded and trampled upon?"

Pope John Paul II, Evagelium Vitae p26, Liberia Editrice 1995.

Asia

"The promotion of sustained economic development is the 'single most important target' for the United Nations". International peace and stability will be enhanced only when all countries enjoy a minimum of standard of economic self-sufficiency and well-being."

Indian Prime Minister Inder Kumar Gujral, Annual General assembly debate in 1997.

G-8 Summit Communiqué

"Our challenge is to build on and sustain the process of globalization and to ensure that its benefits are wide spread more widely to improve the quality of life of people everywhere. Of the major challenges facing the world on the threshold of the 21st century this summit has focused on three: Achieving sustainable economic growth and development throughout the world in a way which will enable developing countries to grow faster and reduce poverty, restore growth to emerging Asian economies and sustain the liberalization of trade in goods and services and of investments in a stable international economy.

Building lasting growth in our economies in which all can participate, creating jobs and combating social exclusion:

In an interdependent world, we must work to build sustainable economic growth in all countries."

This book provides the tools needed to understand and do research in the many of the applied areas cited. It also provides a basis for the computer aided numerical work which will help in forecasting and prediction. For example the theoretical implication in Section 10.4, Theorem 10.4.1, Theorem 10.4.3 for economic policy is now explicitly evident from the models: the solidarity function q=[T,g,e,d,M1,M1,fo], where q is defined in (1.10.75). The possibilities of solidarity, Q, the solidarity set, should be dominated by P the possibilities of private initiative, p=[Co,Io,Xo,Mo,n,w,x0,yo,po] defined (1.10.76). The controllability theorems may now be understood: they give conditions which will ensure that the initial economic state of gross national product, interest rate, employment, value of capital stock, prices (and cumulative balance of payment can be controlled to another state simultaneously subject to scarcity. The MATLAB program needed is detailed and can easily be mastered through the U.S.A. example.

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Preface to the Second Edition

The excellent review and favorable reception of the first edition and the recent G-8 affirmation of the great importance of economic growth of nations, (an application of the theory treated in this book) have inspired the author to bring out this new edition. Some minor errors contained the first edition are corrected. More detailed economic model of the growth of wealth of nations is presented as an application of hereditary systems theory. This second edition contains section 1:10 which details a careful derivation of a neutral differential dynamics as the growth of wealth of nations. It contains a computer program, an appendix that confronts the economic data of the U.S.A. and the model dynamics by identifying its coefficients. This inclusion is motivated by the recent universal opinion of the world leaders on the need for and the importance of better national economic models.