

# Contents

Dedication	v
Acknowledgment (First Edition)	vii
Acknowledgment (Second Edition)	ix
Preface	xv
Preface to the Second Edition	xix
<b>Chapter 1 Examples of Control Systems Described by Functional Differential Equations</b>	<b>1</b>
1.1 Ship Stabilization and Automatic Steering . . . . .	1
1.2 Predator-Prey Interactions . . . . .	3
1.3 Fluctuations of Current . . . . .	6
1.4 Control of Epidemics . . . . .	7
1.4.1 First model . . . . .	7
1.4.2 Control of epidemics: AIDS . . . . .	9
1.5 Wind Tunnel Model; Mach Number Control . . . . .	14
1.6 The Flip-Flop Circuit . . . . .	15
1.7 Hyperbolic Partial Differential Equations with Boundary Controls . .	19
1.8 Control of Global Economic Growth . . . . .	21
1.9 The General Time-Optimal Control Problem and the Stability Problem	26
1.9.1 The stability problem . . . . .	27
1.10 Economic Models with Delay . . . . .	28
1.10.1 Prices . . . . .	33
1.10.2 Balance of payment . . . . .	35
1.10.3 Employment and capital stock . . . . .	36

<b>Chapter 2</b>	<b>General Linear Equations</b>	<b>49</b>
2.1	The Fundamental Matrix of Retarded Equations . . . . .	49
2.1.1	Using (2.1.10a) and (2.1.10b) . . . . .	55
2.1.2	Using (2.1.8) . . . . .	57
2.2	The Variation of Constant Formula of Retarded Equations . . . . .	58
2.3	The Fundamental Solution of Linear Neutral Functional Differential Equations . . . . .	60
2.4	Linear Systems Stability [4] . . . . .	65
2.4	Examples . . . . .	70
2.5	Perturbed Linear Systems Stability . . . . .	71
<b>Chapter 3</b>	<b>Lyapunov–Razumikhin Methods of Stability in Delay Equations</b>	<b>79</b>
3.1	Lyapunov Stability Theory . . . . .	79
3.2	Razumikhin-Type Theorems . . . . .	84
3.3	Lyapunov–Razumikhin Methods of Stability in Delay Equations . . . .	86
<b>Chapter 4</b>	<b>Global Stability of Functional Differential Equa- tions of Neutral Type</b>	<b>97</b>
4.1	Definitions . . . . .	97
4.2	Lyapunov Stability Theorem . . . . .	99
4.3	Perturbed Systems . . . . .	104
4.4	Perturbations of Nonlinear Neutral Systems . . . . .	110
<b>Chapter 5</b>	<b>Synthesis of Time-Optimal and Minimum-Effort Control of Linear Ordinary Systems</b>	<b>113</b>
5.0	Control of Ordinary Linear Systems . . . . .	113
5.1	Synthesis of Time-Optimal Control of Linear Ordinary Systems . . . .	114
5.2	Construction of Optimal Feedback Controls . . . . .	118
5.3	Time-Optimal Feedback Control of Nonautonomous Systems . . . . .	125
5.3.1	The underdamped case ( $\alpha^2 - \omega^2 < 0$ ) . . . . .	138
5.3.2	Critically damped system ( $\alpha^2 - \omega^2 = 0$ ) . . . . .	143
5.3.3	Forming the switching curves . . . . .	148
5.3.4	Terminal manifolds and switching locus . . . . .	148
5.4	Synthesis of Minimum-Effort Feedback Control Systems . . . . .	154
5.5	General Method for the Proof of Existence and Form of Time-Optimal	162
5.5.1	Remarks on the assumptions . . . . .	169
<b>Chapter 6</b>	<b>Control of Linear Delay Systems</b>	<b>187</b>
6.1	Euclidean Controllability of Linear Delay Systems . . . . .	187
6.2	Linear Function Space Controllability . . . . .	191
6.3	Constrained Controllability of Linear Delay Systems . . . . .	195

<b>Chapter 7</b>	<b>Synthesis of Time-Optimal and Minimum-Effort</b>	
	<b>Control of Linear Delay Systems</b>	<b>201</b>
7.1	Linear Systems in Euclidean Space . . . . .	201
7.2	Geometric Theory and Continuity of Minimal Time Function . . . . .	210
7.3	The Index of a Control System . . . . .	216
	7.3.1 Applications . . . . .	217
7.4	Time-Optimal Feedback Control of Autonomous Delay Systems . . . . .	228
7.5	Minimum-Effort Control of Delay Systems . . . . .	237
7.6	Proof of Minimum-Effort Theorems . . . . .	248
7.7	Optimal Absolute Fuel Function . . . . .	256
7.8	Control of Nonlinear Functional Differential Systems with Finite Delay: Existence, Uniqueness, and Continuity of Solutions . . . . .	265
7.9	Sufficient Conditions for the Existence of a Time-Optimal Control . . . . .	268
7.10	Optimal Control of Nonlinear Delay Systems with Target in $E^n$ . . . . .	274
7.11	Optimal Control of Delay Systems in Function Space . . . . .	276
7.12	The Time-Optimal Problem in Function Space . . . . .	284
<b>Chapter 8</b>	<b>Controllable Nonlinear Delay Systems</b>	<b>289</b>
8.1	Controllability of Ordinary Nonlinear Systems . . . . .	289
8.2	Controllability of Nonlinear Delay Systems . . . . .	294
8.3	Controllability of Nonlinear Systems with Controls Appearing Linearly . . . . .	306
<b>Chapter 9</b>	<b>Control of Interconnected Nonlinear Delay Dif-</b>	
	<b>ferential Equations in <math>W_2^{(1)}</math></b>	<b>321</b>
9.1	Introduction . . . . .	321
9.2	Nonlinear Systems . . . . .	324
9.3	General Nonlinear Systems . . . . .	328
9.4	Examples . . . . .	332
9.5	Control of Global Economic Growth . . . . .	337
	9.5.1 Introduction . . . . .	337
	9.5.2 Preliminaries . . . . .	338
	9.5.3 Main results . . . . .	339
	9.5.4 Universal laws for the control of global economic growth . . . . .	346
9.6	Effective Solidarity Functions . . . . .	348
<b>Chapter 10</b>	<b>The Time-Optimal Control of Linear Differen-</b>	
	<b>tial Equations of Neutral Type</b>	<b>353</b>
10.1	The Time-Optimal Control Problem of Linear Neutral Functional Systems . . . . .	353
10.2	Forcing to Zero . . . . .	361
10.3	Normal and Proper Autonomous Systems . . . . .	363
10.4	Pursuit Games and Time-Optimal Control Theory . . . . .	367

10.5 Applications and Economic Growth . . . . . 374

10.6 Optimal Control Theory of Linear Neutral Systems . . . . . 377

10.7 The Theory of Time-Optimal Control of Linear Neutral Systems . . . 384

10.8 Existence Results . . . . . 388

10.9 Necessary Conditions for Optimal Control . . . . . 394

10.10 Normal Systems . . . . . 397

10.11 The Geometric Theory of Time-Optimal Control of Linear Neutral  
Systems . . . . . 398

10.12 Continuity of the Minimal-Time Functions . . . . . 401

10.13 The Index of the Control System . . . . . 406

10.14 Examples . . . . . 407

**Chapter 11 The Time-Optimal Control Theory of Non-  
linear Systems of Neutral Type . . . . . 413**

11.1 Introduction . . . . . 413

11.2 Existence, Uniqueness, and Continuity of Solutions of Neutral Systems 415

11.3 Existence of Optimal Controls of Neutral Systems . . . . . 423

11.4 Optimal Control of Neutral Systems in Function Space . . . . . 429

**Chapter 12 Controllable Nonlinear Neutral Systems . . . . . 443**

12.1 General Nonlinear Systems . . . . . 443

12.2 Nonlinear Interconnected Systems . . . . . 454

12.3 An Example: A Network of Flip-Flop Circuits . . . . . 458

**Chapter 13 Stability Theory of Large-Scale Hereditary Systems . . . . . 463**

13.1 Delay Systems . . . . . 463

13.2 Uniform Asymptotic Stability of Large-Scale Systems of Neutral Type 468

13.3 General Comments . . . . . 471

**Appendix to Section 1.10 . . . . . 473**

**Index . . . . . 493**