Chapter 1

Introduction

In a series of seminal papers [3, 4], Pecora and Carroll show how two chaotic systems can be synchronized by decomposing the system into two subsystems. A chaotic system $\dot{x} = f(x)$ is decomposed by the state decomposition $x = (v, u)^T$ into:

$$\dot{x} = \begin{pmatrix} \dot{v} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} f_1(v, u) \\ f_2(v, u) \end{pmatrix} \tag{1.1}$$

The above equation constitutes the *drive* or *master* system. The *response* system* is derived by restricting the system to the component u, i.e. $\dot{w} = f_2(v, w)$. This decomposition is illustrated in Fig. 1.1. If the Lyapunov exponents of the system $\dot{w} = f_2(v, w)$ given a drive signal $v(t)$ from the chaotic system are all negative, then $|w(t) - u(t)| \to 0$ and we say that the response system *synchronizes* to the drive system. Since the trajectories of

*Also called the *driven* or *slave* system.

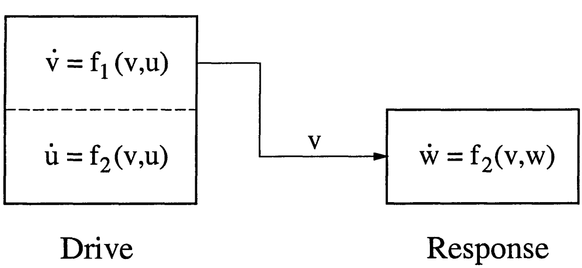

Fig. 1.1 Pecora-Carroll system decomposition.

chaotic systems are in general not periodic, the definition of synchronization used here differs from the classical definitions of synchronization used for coupled periodic oscillators. For chaotic attractors which exhibit a strong periodic component such as in the Rössler system, a generalization of the classical definition is possible [5].

Given the definition of asymptotical stability (Def. C.1), synchronization is equivalent to the fact that $\dot{w} = f_2(v(t), w)$ is asymptotically stable for all $v(t)$ where $x = (v, u)$ are trajectories of $\dot{x} = f(x)$. The main theme of this book is to explore this connection between stability and synchronization and show how stability results can be used to derive synchronization criteria of coupled nonlinear circuits and systems. We mainly focus on analytical techniques such as Lyapunov's direct method and numerical techniques such as the calculation of Lyapunov exponents. Circuit theoretical ideas are introduced whenever appropriate to illustrate their usefulness in studying synchronization in coupled circuits. For an introduction to circuit theory, the reader is referred to [6].

All the circuits and systems we consider in this book are of the form $\dot{x} = f(x, t)$ for the continuous-time case and $x(k + 1) = f(x(k), k)$ for the discrete-time case. We assume that we have existence and uniqueness of trajectories for all time.