

Fig. 1.1 Invariant probability density of a chaotically quantized real scalar field ϕ . The ‘noise’ evolves as $\eta_{n+1} = 2\eta_n^2 - 1$. The mass parameter is given by $m^2\tau = 0.05$.

1.4 * Chaotic quantization in momentum space

Alternatively, we may perform chaotic quantization in momentum space. Let us illustrate this again for the example of the free Klein-Gordon field. We may Fourier transform eq. (1.16) to obtain

$$\frac{\partial}{\partial t} \tilde{\phi}(k, t) = -(k^2 + m^2) \tilde{\phi}(k, t) + \tilde{L}_{l,\tau}(k, t). \quad (1.27)$$

Here $\tilde{\phi}(k, t)$ is the 4-dimensional Fourier transform of the field $\phi(x, t)$,

$$\tilde{\phi}(k, t) = \frac{1}{(2\pi)^2} \int d^4x e^{-ikx} \phi(x, t), \quad (1.28)$$

and $\tilde{L}_{l,\tau}(k, t)$ denotes the 4-dimensional Fourier transform of the chaotic noise field $L_{l,\tau}(x, t)$,

$$\tilde{L}_{l,\tau}(k, t) = \frac{1}{\tilde{\sigma}(2\pi)^2} \tau^{1/2} l^2 \int d^4x e^{-ikx} L_{l,\tau}(x, t) \quad (1.29)$$

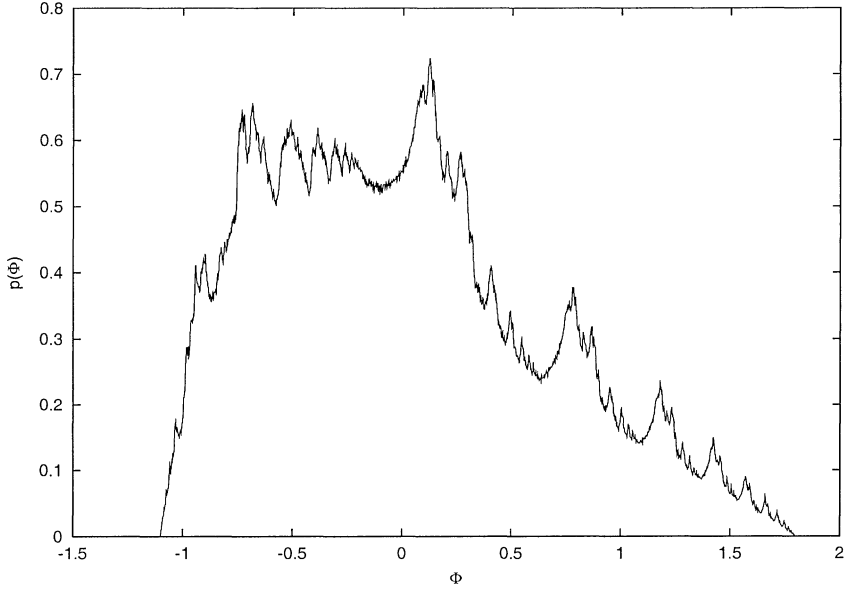


Fig. 1.2 Same as Fig. 1.1, but $m^2\tau = 0.5$.

$$= \frac{1}{\bar{\sigma}(2\pi)^2} \sum_n \sum_j e^{-iklj} \eta_n^j \delta(t - n\tau). \quad (1.30)$$

In k -space the propagator reads

$$G(k, t) = e^{-(k^2+m^2)t}, \quad (1.31)$$

and we obtain

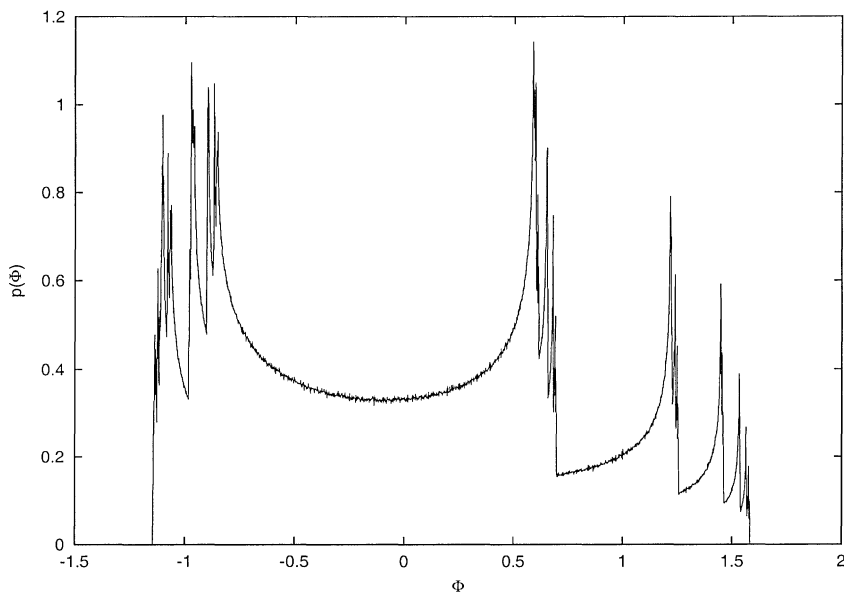
$$\tilde{\phi}(k, t) = \sum_{n=0}^{\lfloor t/\tau \rfloor} b_n(k, t) \tilde{\eta}_n^k, \quad (1.32)$$

with

$$b_n(k, t) := \bar{\sigma}^{-1} \tau^{1/2} l^2 e^{-(k^2+m^2)(t-n\tau)} \quad (1.33)$$

$$\tilde{\eta}_n^k := \frac{1}{(2\pi)^2} \sum_j e^{-iklj} \eta_n^j. \quad (1.34)$$

Of course, under normal circumstances the length scale l is so small that there are no differences compared to the usual stochastic quantization

Fig. 1.3 Same as Fig. 1.1, but $m^2\tau = 1$.

scheme in momentum space. This means, on large scales the Fourier transformed chaotic noise will satisfy the ordinary relations of Fourier transformed Gaussian white noise [Damgaard et al. (1988)]

$$\langle \tilde{L}(k, t) \rangle = 0 \quad (1.35)$$

$$\langle \tilde{L}(k, t) \tilde{L}(k', t') \rangle = 2\delta^4(k + k')\delta(t - t'). \quad (1.36)$$

Differences between chaotic and ordinary stochastic quantization only occur on a very small scale.

1.5 * Gauge fields with chaotic noise

Let us consider another simple example. For a free Maxwell field $A_\mu(x, t)$ (describing photons) the chaotically quantized field equation reads

$$\dot{A}_\mu(x, t) = \partial_\nu F_{\mu\nu}(x, t) + L_{l, \tau \mu}(x, t), \quad (1.37)$$