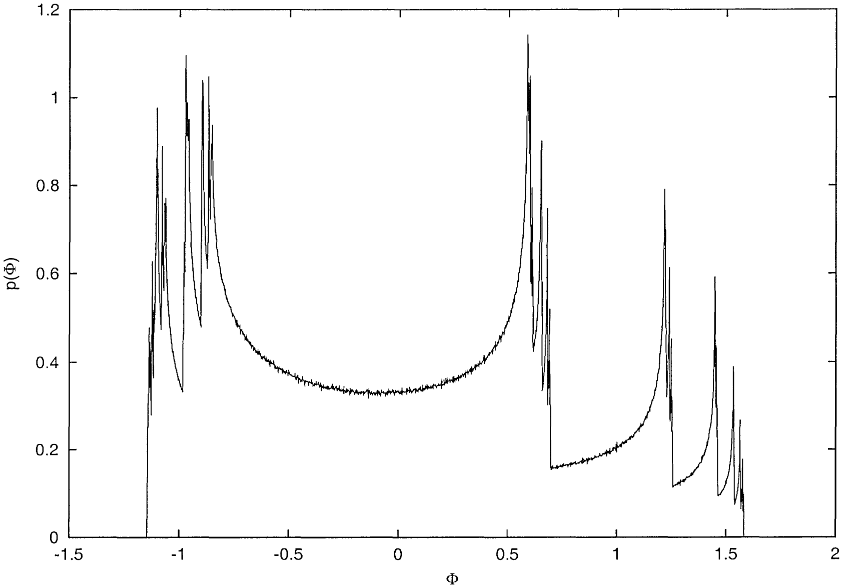

Fig. 1.3 Same as Fig. 1.1, but $m^2\tau = 1$.

scheme in momentum space. This means, on large scales the Fourier transformed chaotic noise will satisfy the ordinary relations of Fourier transformed Gaussian white noise [Damgaard et al. (1988)]

$$\langle \tilde{L}(k,t) \rangle = 0 \tag{1.35}$$

$$\langle \tilde{L}(k,t)\tilde{L}(k',t') \rangle = 2\delta^4(k+k')\delta(t-t'). \tag{1.36}$$

Differences between chaotic and ordinary stochastic quantization only occur on a very small scale.

1.5 * Gauge fields with chaotic noise

Let us consider another simple example. For a free Maxwell field $A_\mu(x,t)$ (describing photons) the chaotically quantized field equation reads

$$\dot{A}_\mu(x,t) = \partial_\nu F_{\mu\nu}(x,t) + L_{l,\tau\ \mu}(x,t), \tag{1.37}$$

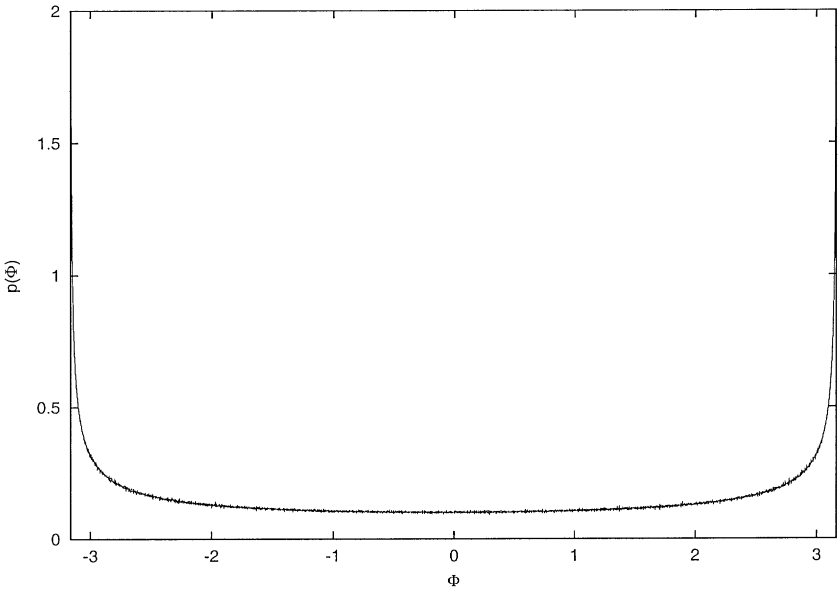

Fig. 1.4 Same as Fig. 1.1, but $m^2\tau = 10$.

with

$$F_{\mu\nu}(x) := \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x). \tag{1.38}$$

We now need 4 independent components $L_{l,\tau\ \mu}(x,t)$, $\mu = 1,2,3,4$, of chaotic noise. Again it is useful to proceed to the momentum space, where we obtain an equation for the Fourier transform $\tilde{A}_\mu(k,t)$ of the field $A_\mu(x,t)$:

$$\dot{\tilde{A}}_\mu(k,t) = -k^2\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\tilde{A}_\nu(k,t) + \tilde{L}_{l,\tau\ \mu}(k,t). \tag{1.39}$$

It is convenient to decompose the field into a transverse part $\tilde{A}^T_\mu(k,t) := (\delta_{\mu\nu} - k_\mu k_\nu/k^2)\tilde{A}_\nu(k,t)$ and a longitudinal part $\tilde{A}^L_\mu(k,t) := (k_\mu k_\nu/k^2)\tilde{A}_\nu(k,t)$ (see, e.g., [Namiki et al. (1983)]). The quantized field equation can then be written as

$$\dot{\tilde{A}}_\mu{}^T = -k^2\tilde{A}^T_\mu + \tilde{L}^T_{l,\tau\ \mu} \tag{1.40}$$

$$\dot{\tilde{A}}_\mu{}^L = \tilde{L}^L_{l,\tau\ \mu}, \tag{1.41}$$

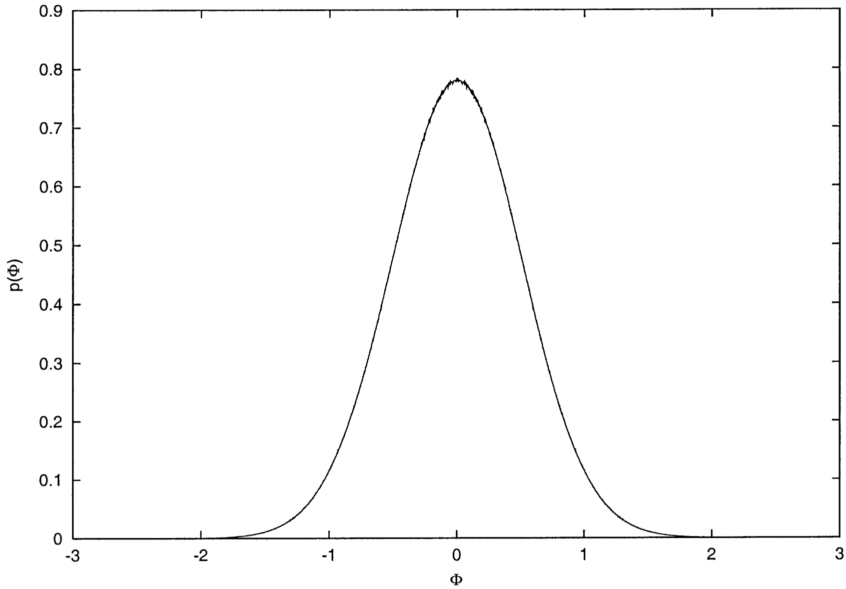

Fig. 1.5 Invariant probability density of a chaotically quantized real scalar field ϕ with 'noise' evolving as $\eta_{n+1} = 4\eta_n^3 - 3\eta_n$. The mass parameter is $m^2\tau = 0.05$.

where $\tilde{L}^T$ and $\tilde{L}^L$ are projections of the chaotic noise onto the transverse, respectively longitudinal direction. This shows that the transverse component of the gauge field behaves like a massless Klein-Gordon field with chaotic noise, and the same consideration as in the previous section applies, this time with $m = 0$. From a stochastic process point of view, the transverse component $\tilde{A}^T_\mu(k,t)$ is a generalized Ornstein-Uhlenbeck process, the Gaussian white noise being replaced by chaotic noise. The longitudinal component $\tilde{A}^L_\mu(k,t)$ is just the time-integrated chaotic noise field, and in this sense it is the chaotic generalization of a Wiener process.

1.6 Distinguished properties of Tchebyscheff maps

Let us now think about a suitable dynamics for the deterministic chaotic noise fields η_n^i. The most distinguished candidate is of course a dynamics that (in some appropriate sense) is closest to Gaussian white noise, though being completely deterministic. As we shall see in this and the following