It follows that we have, with $p_{6i} = \gamma_{6i} = \delta_{6i}$:

$$\begin{aligned} Y_i(t+1) &= \sum_{u=1}^{5} \left\{ \sum_{v=1}^{5} Z_{uv}(i,t) + W_{u6} p_{6i} \right\} + \delta_{i6} I_6(t) \\ &= \sum_{u=1}^{5} \sum_{v=1}^{6} W_{uv}(t) p_{vi} + \delta_{i6} I_6(t) + e_{i1}(t+1) \\ &= \sum_{u=1}^{5} I_u(t) \sum_{v=1}^{6} \gamma_{uv} p_{vi} + \delta_{i6} I_6(t) + e_{i1}(t+1) + e_{i2}(t+1) \\ &= \sum_{u=1}^{6} I_u(t) \sum_{v=1}^{6} \gamma_{uv} p_{vi} + e_i(t+1) \,, \end{aligned}$$

where

$$e_{i1}(t+1) = \sum_{u=1}^{5} \left\{ \sum_{v=1}^{5} [Z_{uv}(i,t) - W_{uv}(t) p_{vi}] \right\} ,$$

and

$$e_{i2}(t+1) = \sum_{u=1}^{5} \left\{ \sum_{v=1}^{6} p_{vi} [W_{uv}(t) - I_u(t) \gamma_{uv}] \right\} ,$$

and $e_i(t+1) = e_{i1}(t+1) + e_{i2}(t+1)$.

Put $P = (p_{ij})$ and $H = P'F$. Denote by $\underset{\sim}{Y}(t) = \{Y_1(t), \ldots, Y_6(t)\}'$ and $\underset{\sim}{e}(t) = \{e_1(t), \ldots, e_6(t)\}'$. Then, in matrix notation, we have:

$$\underset{\sim}{Y}(t+1) = H \underset{\sim}{I}(t) + \underset{\sim}{e}(t+1) \,. \tag{1.4}$$

Equation (1.4) is the observation model for the state space model associated with the above hidden Markov chain.

1.5. The Scope of the Book

The stochastic models described in Secs. 1.1–1.4 are the major models which arise from genetics, cancer and AIDS. In this book we will thus present a systematic treatment of these models and illustrate its applications to genetics, cancer and AIDS.

In Chap. 2, general theories of Markov chains with discrete time will be presented and discussed in detail. As a continuation of Chap. 2, in Chap. 3, we will present some general theories on stationary distributions of Markov chains with discrete time; as an application of stationary distributions, we also present some MCMC (Markov Chain Monte Carlo) methods to develop computer algorithms for estimating unknown parameters and state variables. In Chaps. 4 and 5, general theories of Markov chains with continuous time will be presented and discussed in detail. Applications of these theories to genetics, cancer and AIDS to solve problems in these areas will be discussed and demonstrated. In Chaps. 6 and 7, we will present and discuss in detail general theories of diffusion processes. We will show that most processes in genetics, cancer and AIDS can be approximated by diffusion processes. Hence, one may use theories of diffusion process to solve many problems in these areas. Finally in Chaps. 8 and 9, we present and discuss some general theories of state space models and illustrate its applications to cancer and AIDS.

This book is unique and differs from other books on stochastic processes and stochastic models in that it has presented many important topics and approaches which would not be discussed normally in other books of stochastic processes. This includes MCMC methods and applications, stochastic difference and differential equation approaches to Markov chains as well as state space models and applications. It follows that there are minimal overlaps with other books on stochastic processes and stochastic models. Also, the applications to cancer, AIDS and genetics as described in this book are unique and would normally not be available in other books of stochastic processes.

1.6. Complements and Exercises

Exercise 1.1. Let $\{X(t), t \geq 0\}$ be a stochastic process with state space $S = (0, 1, \ldots, \infty)$. Suppose that the following two conditions hold:

(a) $P\{X(0) = 0\} = 1$.

(b) $\{X(t), t \geq 0\}$ has independent increment. That is, for every n and for every $0 \leq t_1 < \cdots < t_n$, $\{Y_j = X(t_j) - X(t_{j-1}), j = 1, \ldots, n\}$ are independently distributed of one another.

Show that $X(t)$ is a Markov process.

Exercise 1.2. Let $\{X(j), j = 1, 2, \ldots, \infty\}$ be a sequence of independently distributed random variables. That is, for every n and for every set of integers