

Chapter 4

The dream of a hierarchical world: protofractals

The 18th century was the time of the Enlightenment, when every observed phenomenon, from the human body to the clockwork of the universe, became amenable to scientific scrutiny and explanation. By the “siècle des lumières” the luminaries of the night sky had scattered into the depths of space. But what is the shining band of the Milky Way?

One clear night Johann Lambert stared at the sky, hoping to see some order: “Last night I again inspected the starry sky, as I have never been able to find any definite *symmetry* in its appearance. Once again in vain. Then I noticed that the stars of first, second, and third magnitude are very unevenly distributed, somewhere densely together, while elsewhere in the sky there are large empty spaces hardly containing a few stars of sixth magnitude. So, I thought, would for us appear the solar *system*, if we could see all the planets and comets at the same time.”

What a wonderful night! Structure was discerned in the sky, telling about the stars in *space*. Maybe they form a large, flattened system similar to our swarm of planets around the Sun? At the time of Lambert, the third dimension, depth in space, achieved a new fascination.

* * *

4.1 Stars and nebulae

Isaac Newton regarded it as highly desirable that stars be uniformly distributed in infinite space. Otherwise it was hard to understand the peaceful scenery where we are living, not having collapsed into a huge mass nor feel-

ing intolerable forces emanating from the abysses of space. Uniform matter was needed to cancel the gravity force. But the real stars in the sky posed a problem. The most conspicuous feature of the stellar vault is the Milky Way. Galileo had seen through his magnifying tube that its pearly glow comes from innumerable faint stars.

Newton tried to show that if one goes deeper and deeper into space, i.e. looks at fainter stars, then uniformity is encountered. Modern cosmologists have a similar hope: looking at large enough parts of the universe, one should finally see a uniform world of galaxies (the new basic units of matter in the large). Newton's attempts remained inconclusive. This is not surprising. Nearby stars actually form a large flattened cloud. Newton also did not know how the magnitudes of stars depend on their distances, and hence how many faint stars there should be, in comparison with the bright ones, if stars are uniformly dispersed in space.

By the time of Newton, keen observers had begun to look with their telescopes also at other things than stars. In the 18th century, comet hunting became popular. However, not all diffuse blobs in the sky are comets. There are a lot of immobile nebulae which annoyed the comet hunters because one easily confused them with the slow moving comets. Charles Messier decided to collect a useful catalog of such nuisances. During his career Messier discovered some twenty new comets and was invited to become a member of the French Academy. Though, he is now remembered less for his misty comets than for his nebula catalog which contained 103 objects, their positions in the sky and descriptions of their appearance. Astronomers still refer to M31 (or Messier 31) when they speak about the Andromeda nebula. Also many other beauties of the night are known by their Messier-numbers. Though Messier was not at all interested in nebulae *per se*, some minds started to wonder what they are.

*Charles
Messier
1730-1817*

4.2 Emanuel Swedenborg

*Emanuel
Sweden-
borg
1688-1772*

Emanuel Swedenborg was a Swedish scientist and visionary. His father was a professor at Uppsala University and later the bishop of Skara. The young Emanuel studied languages and natural sciences and after graduation in 1709 he embarked on travels in Europe. There he became acquainted with the scientific circles of the time and built a reputation as a skilled engine constructor. After returning home, he worked as the editor of Sweden's first

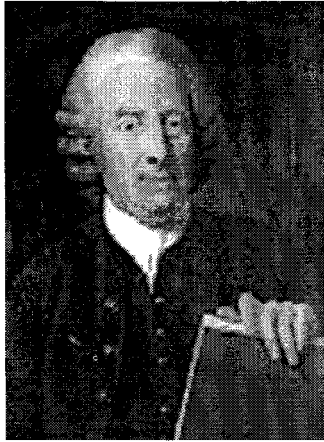


Fig. 4.1 Emanuel Swedenborg expressed his principle of self-similarity in different ways. One was “Nature seems to admire her own charms”.

scientific periodical and was appointed, at the age of 28, assessor extraordinary in the College of Mines. This high position he held for over thirty years. He was also the corresponding member of the Imperial Academy of Sciences of St. Petersburg. The remains of this unusual man rest in the cathedral of Uppsala.

Swedenborg was a productive thinker and writer, who discussed practically all fields of science of his time and wrote treatises in physiology, zoology, chemistry, geology, mineralogy, physics, and astronomy. * He appears in our book because of his ideas on the structure of the universe. In 1734, in his *Principia*, Swedenborg put forward the remarkable views of self-similarity and cosmic hierarchy: elementary particles form celestial bodies which form systems which in their turn may be elements of systems higher in the hierarchy, and so on. This view reflected his general opinion that everything in the world is constructed according to a common plan.

Swedenborg's work in science and engineering has often been overshadowed by the last three decades of his life when he studied things which are rather strange to us: he became a clairvoyant, had conversations with spirits and angels, and wrote books on the spiritual world and theology.

*For instance, it is nowadays recognized that Swedenborg's writings on the brain and sense organs were “observationally more accurate and theoretically more profound than those of any other eighteenth-century scholar” (cited from *Nature* 394, p. 144 (1998)).

But remember that the great alchemist Newton devoted a majority of his time on other than what we now call scientific activities – the age when Newton and Swedenborg lived was different from ours...

4.3 Cartesian physics

*René
Descartes
1596-1650*

Swedenborg was much influenced by the ideas of René Descartes. This French mathematician, physicist, and philosopher paved the way for modern science at a time when it was still not always healthy to speak about Copernicus. Descartes had a considerable family fortune, which allowed him to combine scientific pursuits with travel around Europe. He lived and died outside his native country, residing twenty years in the more peaceful and liberal Netherlands. The last months of his life he spent in Stockholm where he was invited by Queen Christina. The chilly Nordic winter and the very early morning philosophical teachings ordered by the queen were too much for his health, which had always been fragile.

In fact, already during his school years the frail boy was advised to lie in bed as late as he pleased in the mornings. There is a story that thanks to this lifelong habit (which he combined with thinking) he invented analytical geometry. One morning his eye caught a fly crawling on the ceiling of his bedroom. How could one describe the path of the fly mathematically? The answer was given by x and y coordinates: in his imagination Descartes labeled each point of the ceiling by an (x, y) pair of numbers. Geometry and algebra became happily married. And the matchmaker came to be called the father of modern mathematics.

In 1619 Descartes experienced three dreams which he interpreted as an invitation to reconstruct and raise human knowledge to the level of certainty possessed heretofore only by mathematics. One result was a doctrine, the Cartesian system. It enjoyed high popularity in the seventeenth and eighteenth centuries, but also was attacked by the Church (both Catholic and Protestant) as poorly disguised Copernicanism.

A starting point of Cartesian physics was the law of inertia, previously discussed by Galileo, but which Descartes clearly formulated for a particle residing in an infinite universe. Without contact with other particles, it either would keep its initial state of rest or would move with a constant speed along a straight line, until deflected by a collision with another particle. In the light of this principle (in which one recognizes Newton's

subsequent first law of motion), the various changing movements in the real world are caused by some impact. There is no vacuum and no mysterious action-at-distance. Bodies are all the time in contact with other bodies.

Descartes interpreted visible phenomena in terms of microscopic interactions. So, the attraction of a magnet for a piece of iron would be caused by invisible screw-shaped particles which are emitted from the magnet and enter screwed channels existing in the iron. The motions of the planets are forced by an ethereal vortex around the sun, somewhat as bits of cork are caught up in a whirlpool. Similar perpetual vortexes exist around other stars. The space between stars is not empty, but filled by particles of the ocean of ether.

4.4 The Swedenborg self-similar universe

Swedenborg was well versed in Newton's theory which had not yet been known in Descartes's lifetime, and which was destined to replace Cartesianism. He even brought to Sweden the mathematics of the new physics, the calculus. However, he was attracted by the basic ideas of Descartes (material interactions, vortices, no vacuum) and attempted to draw a world picture on this foundation. He was guided by a particular assumption: that everything in the world, the small as well as the great, is made according to similar principles. This idea of self-similarity is the golden thread in Swedenborg's *Principia*. On it he bases his attempt to build a consistent theory of particles and celestial bodies. He was fascinated by the possibility to find by a scientific method ("experience, geometry, rational reasoning") the secrets of the invisible world of elementary particles.

Swedenborg was much interested in the properties of magnetism, especially the concentrations of iron particles along, as they are now called, lines of force. The concentration is heaviest close to the axis of a magnet. Because magnetic force is much stronger than gravity, one might see a signature of its cosmic analogy also in the structure of Heavens: "... and from a vile stone of the earth and its magnetic powers, contemplate what is similar on the largest scale".

Dante wrote in his *Divina Commedia*: "Pricked out with less and greater lights, between the poles of the universe, the Milky Way so gleameth white as to set very sages questioning". The Swedish sage paid attention to the fact that stars are packed most closely along the Milky Way and reasoned

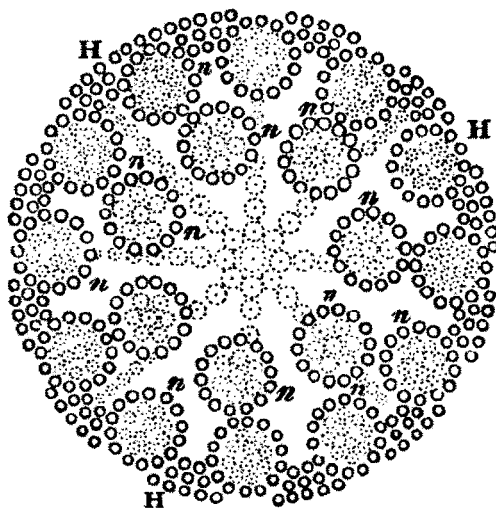


Fig. 4.2 Swedenborg's model of elementary particles. The surface of the particle is made of smaller particles which inside the particle form identical copies of the latter.

that this should correspond to an axis in the system of the stars. Swedenborg's text suggests that he pictured the Milky Way as a straight column in space. A less likely interpretation is that he described a ring (like the circular lines of force around a ring-shaped magnet). In any case, he was perhaps the first to realize that what we can see projected in the sky, may offer information on the large scale distribution of matter.

The axis of our Solar System, which also is a big magnet, deviates from the cosmic axis defined by the Milky Way. This means that we reside somewhat aside from the main concentration, and so we can see it as a column in the sky, according to Swedenborg.

Swedenborg viewed the microcosm as formed by progressively smaller and smaller particles. But the series does not extend to infinitely small sizes or arbitrarily fine substances. There is the "first finite" or the substance where "geometry begins", born from the "natural point", which is a kind of singular state between the unexplorable Infinite and our world.

There is no vacuum and no independent space (as Newton postulated), but the concept of space refers to relations between particles. Looking at the macrocosm, the world of planets and stars, Swedenborg extended to

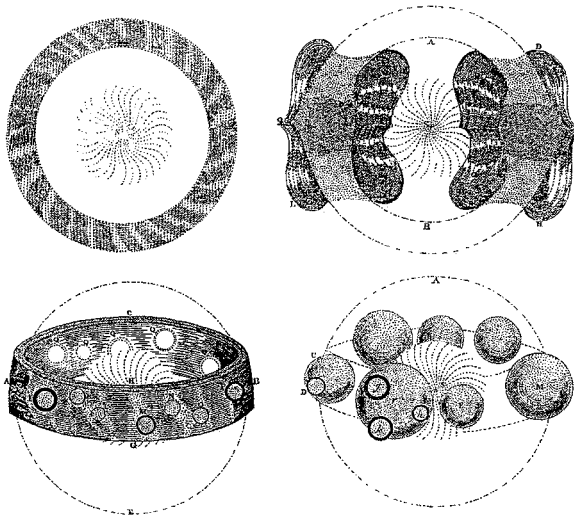


Fig. 4.3 Swedenborg's view of how the planets were born by condensation from a ring of matter which collapsed upon itself around the primordial Sun. The revolving ring explains why the planets are in a same plane and orbit in the same direction.

those large scales the multilevel structure which he imagined to exist in the microcosm. The Milky Way, formed by other suns, is just one element of a larger system which is an element of a still larger one, and so forth. This grandiose view was some decades later taken up by Kant and Lambert.

Time has almost forgotten the ambitious world architecture of Swedenborg, based on magnetism rather than gravity, and ignoring the then unknown quantum laws of the microworld. Nevertheless, his *Principia* was a result of much penetrating thinking and is still fascinating reading.

4.5 Towards the origin of the Solar System

Emanuel Swedenborg presented a sketch for the birth of planetary systems (of which ours was known, and others assumed). The origin of the flat Solar System, with the planets revolving on almost circular orbits, appeared as a great enigma in Newton's letters to Bentley. Descartes had thought that planets had previously been wandering stars in the cosmic space between other stars. After they were extinguished, their ethereal vortex had weakened and they were threatened by the attraction of the Sun's whirlpool.

With fine intuition and assisted by the idea of self-similarity, Swedenborg found it simpler to imagine that the origin of the planets and moons lies in the primordial solar mass itself. He suggested that the swift rotation of the solar mass makes, as a result of centrifugal force, its outermost parts to be cast out.

He then proposed that the planets are formed by condensation out of the detached ring of matter. Later Kant and Laplace presented a roughly similar picture for the origin of planets, already dressed in Newtonian clothing, the celebrated Kant–Laplace nebular hypothesis. These old ideas are still a part of our astronomical world view, even if the physical processes turned out to be more complicated.

Swedenborg even pointed out evidence for such an evolutionary process: the starry sky is not quite unchanging. Sometimes a new star is born and then vanishes. This happens when the broad belt of condensed matter covers the young star from sight. Now we know that such exploding and dimming new stars actually are a late phase in the life of massive stars.

*Georges
Buffon
1707-1788*

It is often said that the French scientist Georges Buffon was the first to present a scientific theory of the formation of the Solar System. However, his theory, based on the idea of material torn out of the Sun by a comet, was published ten years after Swedenborg's *Principia*. †

4.6 Hierarchies of Kant and Lambert

Immanuel Kant and Johann Lambert both also pondered the appearance of the night sky above their respective native towns Königsberg and Mülshausen (the latter in Swiss Sundgen, at present French Alsace).

In a letter written by the mathematician and physicist Lambert to the philosopher Kant, one can read how a deep though simple idea may be born: “contrary to my habits then, I went into my room after the evening meal, and looked through the window at the stellar sky, and especially at the Milky Way. The insight, which I had then, to see it as an ecliptic of the fixed stars, I wrote down on a quarto page”

†*Principia* received the honor of being placed on the Index of the Catholic Church. The translator of *Principia*, the Rev. Augustus Clissold speculates that this happened because the proposed formation of the Solar System appeared to oppose the doctrine that God created all things out of nothing and was also difficult to reconcile with the literal interpretation of the first chapter in Genesis.

In his *Cosmological Letters*, Lambert describes the thoughts which the Milky Way inspired. He marveled at the great number of faint stars in that narrow strip across the sky. He found it hard to believe that they could lie tightly packed side by side at a similar distance from us. They should be distributed in depth, and the brightness of the Milky Way said to him that in that direction the rows of stars must be much deeper than outside it. “Briefly said, the edifice of stars is not spherical, but flat, even very flat.”

*Johann
Lambert
1728-1777*

At about the same time as Lambert inspected the sky (1749), the self-taught theologian and scientist Thomas Wright arrived at a remarkable view of the heavens. For years he had aimed at making a world model which would contain God and which would explain the appearance of the starry sky. For the latter purpose he imagined that the Sun is situated in the middle plane of a layer of stars. When one looks in the direction of the layer, many stars are seen, similarly as Lambert thought. However, Wright preferred something greater than a flat cloud. He pictured a huge spherical shell formed by stars which revolve around a distant center containing a “Primum Mobile”, a large gravitating mass. The center is also the mansion of God. If the stellar shell is relatively thin, then in the vicinity of the Sun it is almost planar up to some distance, producing the great circle of the Milky Way on the sky. Wright also regarded it as probable that there are other similar Milky Ways, “many cloudy spots, just perceivable by us”.

*Thomas
Wright
1711-1786*

Lambert’s and Wright’s reasoning about the strip of the Milky Way in the starry sky is a nice example of how a natural phenomenon, visible to everyone from times immemorial, gains new significance, literally a new dimension, after a bright flash of idea.

Kant happened to read a newspaper report which described a book written by Wright. This gave Kant food for thought, and he also drew a cosmological picture where the Milky Way is a flattened cloud of stars and the Sun is a member of this cloud. Further, similarly as Lambert and Wright did, he presented the hypothesis that the pale elliptical nebulae, observed by astronomers, are other Milky Ways. Their great distances do not permit us to see they consist of stars.

*Immanuel
Kant
1724-1804*

Kant and Lambert also suggested that the stellar systems form a hierarchical structure, so they are not distributed uniformly. Kant wrote: *It might further be conjectured that these higher universes are not without relation to one another, and that by this mutual relationship they constitute again a still more immense system . . . which perhaps, like the former, is yet again but one member in a new combination of numbers! We see the first*

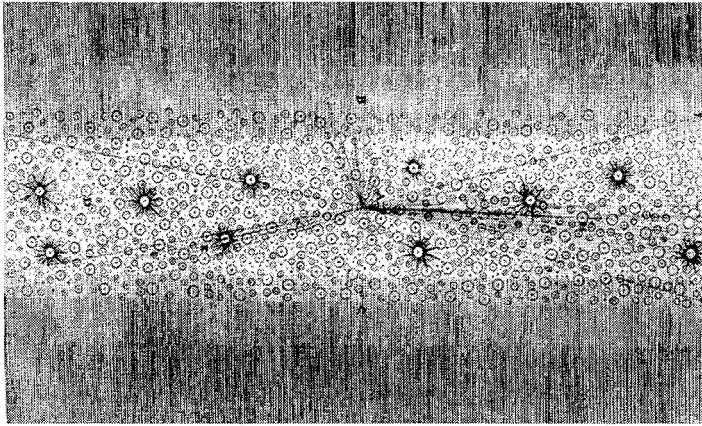


Fig. 4.4 Thomas Wright explained how a thin layer of stars gives rise to a strip of stars on the sky, the Milky Way. Imagine yourself placed in the middle of the layer.

members of a progressive relationship of worlds and systems; and the first part of this infinite progression enables us already to recognize what must be conjectured of the whole. There is no end but an abyss . . . without bound.

Swedenborg wrote about his cosmic spheres: *That there may be innumerable spheres or sidereal heavens in the infinite universe, that these may be colligated one with the other, like the spheres of two magnets; and the whole visible sidereal heaven is perhaps but a point in respect to the universe.*

4.7 Finite or infinite?

There was a difference between these world models. Kant, and perhaps Swedenborg, imagined that the hierarchy continues without end towards larger and larger levels of celestial systems: it was an infinite hierarchy. Lambert thought that after a large (he cites 1000 as an example), but finite number of steps, the hierarchy ends. He thought that the stellar systems are kept together by the gravity of dark stellar masses: “. . . in the end you arrive at the middle point of the whole world structure and there I find my ultimate mass which governs the whole creation.”

In his *Universal Natural History*, Kant explains why there should be a hierarchy of celestial bodies. Universal gravity makes smaller bodies orbit around greater masses. Planets circling the Sun is an example and similarly the stars of the Milky Way should according to Kant revolve around some

mass that resides in the center of the stellar system. Analogously, the Milky Way is an element of a much larger system, again revolving around its center. The universally penetrating gravity keeps both small and large systems tied together, making possible the world structure. Hierarchic system seemed to offer a solution to the problem of stability of the universe, which was so puzzling thing in Newton's uniform stellar world. Stars and their systems do not collapse, but rotate around distant centers.

Kant speculated on how such a hierarchy could have formed. He takes the universe to be spatially infinite and initially filled with a thin substratum. He proposes that some finite time ago there was a moment of creation whereby at some point of the infinite universe there appeared a large mass. This mass, defining the "center" of the universe, put in motion the matter around it and agglomerations of other masses started to occur around the center, this process spreading as a chain reaction farther and farther from the primordial center. The process leads to systems of different order.

4.8 Emerging protofractals

Kant was very excited about this view of a developing matter distribution. However, his discussion shows that the system (similarly to the hierarchy of Lambert) has a preferred center which not everyone is ready to accept as a nice cosmological picture. Such structures may now be termed as *protofractals*, predecessors of the concept of the fractal. † They exhibited self-similarity: the structure in large is repeated in the structure in small.

Already the oldest drawings, as found in caves tens of thousands of years old, testify the pleasure which people have always felt in looking at geometric patterns revealing similarity and symmetry. This affection culminated in the cosmological model based on spheres and circular motion, as studied in Plato's Academy. The world was divided into heavens, planetary spheres, and the sublunary region. When the Jewish philosopher Philo wrote the impressive words: "Heaven alone is unchangeable and self-consistent and similar to itself", he referred, not to fractals (!), but to the spherical heaven.

Philo
c.20 B.C.-
c.50 A.D.

This view was beautifully illustrated by a drawing, made widely known by Benoit Mandelbrot, from a Bible Moralisée written between 1229 and

†The term "protofractal" for such early examples of structures resembling modern fractal was suggested by Benoit Mandelbrot to the authors of this book at a conference in Paris in 1999. The Greek word "protos" means "first" (c.f. elementary particle proton).

1250. There God the Geometer creates from an amorphous mass the “sky and earth, sun and moon and all elements”. The whole world is a sphere, and in its interior Mandelbrot sees “circles, waves, and wiggles”, the first two forms being the subject of most mathematics and science, while the “wiggles” have only in modern times become tractable as fractals.

After Copernicus, the heavenly spheres started to erode. Before Newton’s explanation of planetary motion, a new view of the cosmos and its forces was drawn by Descartes. His vortices made the large scale universe closer connected with our complex immediate neighborhood. One could see nearby analogies of cosmic processes in the eddies of the restless river and in the whirlwinds shaking the trees. Descartes regarded that every star, including our Sun, is in the center of a matter vortex.

Inspired by Descartes’s cosmology of an infinite stellar world, Bernard le Bovier de Fontenelle, a French scholar, bravely went a step further. In his *Conversations on the Plurality of Worlds* of 1686 that passed through 31 editions during the author’s long lifetime, Fontenelle supposed that stars have around them planetary systems, and each planet is also surrounded by a vortex inside the major “turbillon” around the star. These smaller vortices may drive moons around the planets. Fontenelle’s charming book was an early attempt to popularize science. Its concept was simple enough: a man of science walking in the moon-lit garden with a sweet lady, explaining to her the secrets of Nature. He “chose from all Philosophy the topic which most of all arouses curiosity”, i.e. the structure of the world and whether there are other worlds also inhabited by living beings. To the last question Fontenelle answered emphatically “yes”.

Another hierarchy of vortices, though unquestionably real, had been studied by Leonardo da Vinci in his superb drawings of turbulent water. His eye caught eddies inside eddies inside eddies, where someone else saw just a mess of surge and foam. Nowadays fluid “turbulenzia” – the word first used by Leonardo – is a major example of a hierarchic, fractal system.

The painter Ivan Aivazovskij became famous for his sea-theme, which is also interesting for glimpses of the “pre-fractal” view of Nature. The sea is not just a field of regularly spaced waves, but – as a scenery of mountains – big waves are superpositions of smaller waves. The clouds hanging over the troubled waters have edges with details in large and small.

These painters reproduced with their skilfull hand what their acute eyes saw in Nature. Jumping a little ahead in our story, we note that excursions of another type to the “wiggle” world were made by modern abstract painter

Jackson Pollock in his “action painting”. He did not copy Nature, but used its own methods, which produced structures that were later called fractals. We come back to his work in Chapter 14.

4.9 Inwards and outwards

One likes to think that understanding something means an ability to imagine it as a whole. The simplest picture that Man has learnt to draw of the universe is the uniform matter distribution which continues without limits. This seems to be easy to imagine, and non-Euclidean geometry has even waived the need to extend the distribution into infinity.

But why do hierarchies also please the mind as structures of the physical reality? Why were some thinkers attracted by such a non-uniform picture even when observations did not offer evidence? Perhaps because one does not have to travel far away in order to see whole new worlds, in particular if hierarchical levels exist below you. Just change your size, or in practice, the scale of the inspected part of the World. In the words of William Blake:

*To see a World in a Grain of Sand
And a Heaven in a Wild Flower,
Hold Infinity in the palm of your hand
And Eternity in an hour*

*William
Blake
1757-1827*

Or as mathematician Hermann Weyl put it, the space is “inwardly infinite”, thus a potential mansion of rich structures.

The notion that there is a correspondence between the macro- and microcosm, has of course appeared before Swedenborg. So Gottfried Leibnitz entertained this idea in his *Monadology* from 1716: “Every portion of matter may be conceived as like a garden full of plants and like a pond full of fish. But every branch of a plant, every member of an animal, and every drop of the fluids within it, is also such a garden...” A less idyllic scene was described by Bruno (with his tongue in his cheek, certainly). When one of his heros tells that “...from the inspection of the macrocosm it is easy, by making necessary conclusions from the similarity, to learn about the microcosm, the particles of which correspond to the parts of the former.”, the other one replies: “So that we can discover inside You the Moon, Mercurius, and other stars; France, Spain, Italy, England, Calcutta, ...?”

*Gottfried
Wilhelm
Leibnitz
1646-1716*

Swedenborg was guided by the beneficial aspect of self-similarity. Re-

ferring to the different levels of systems, he was happy to say that “he who has learnt the nature of one will have learnt the nature of all”. He took self-similarity as a cosmological principle which permeates the cosmos on its all scales. Only later, when one could observe both the micro- and the macroworld, did it become possible to check how widely this idea applies.

Kant made an interesting prophecy: “. . . the formation of all celestial bodies, the cause of their motions, in brief, the origin of the whole present arrangement of the world edifice, will sooner be understood than the production of a single herb or of a caterpillar will come evidently and completely clarified from mechanical reasons”. Kant discusses the question how a man, who easily errs in the small things around, could be able to study the large scale things. He asserts that among all tasks of the study of nature none can be solved more correctly and with more certainty than the structure of the world edifice at large. Why is he so confident? He points out that celestial bodies populate an empty space and are separated by great distances. There is only the attractive gravity force influencing their motions. All this is much simpler than what makes the things work here on the earth. Thus Kant foresaw that on widely different scales different physical laws dominate – “he who has learnt the nature of one” does not necessarily know the nature of all.

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Isaac Newton, Kant’s master in physics, had speculated that in order to understand small things, one has to assume that down in the microworld there are both attractive and repulsive forces which arrange tiny atoms into molecules and these into larger hierarchic systems up to visible things. Solid crystals and beautifully regular, but shortlived snowflakes indicated for him the presence of such forces. Modern physics has proven true Newton’s idea that the “Hate” of Empedocles works together with his Love in molding a rich physical reality.

The ideas of self-similarity, fractality and chaos became an important part of scientific culture towards the end of the 20th century. But before that happened science made illustrious other advances.