1.6.7 *Basics of PDO theory*

In the last two chapters, we provide the essentials of the theory of PDO used in the book. This is useful for a reader interested in the systematic exposition of technical tools used throughout the book, though we tried to explain the most essential things in the main body of the book. There is another aim of the chapter: a specialist with the PDE-background tends to regard the Black-Scholes equation as a primitive object of Mathematical Finance, and the exposition in Wilmott *et al.* (1993) is a model example of this approach. The basic theory presented in Chapter 15 allows the reader to calculate prices of contingent claims formally, without mentioning stochastic integrals, thereby providing the background for PDE-approach in non-Gaussian situations.

1.7 Commentary

The systematic exposition of the Gaussian theory of Mathematical Finance, at different levels, can be found in many monographs. Duffie (1996) is a very good book on the level intermediate between Economics and Mathematics; Hull (2000) is, probably, the best professional book on derivatives; Karatzas and Shreve (1997) and Shiryaev (1999) are excellent rigorous mathematical texts (in addition, Shiryaev (1999) discusses various fundamental aspects of non-Gaussian Mathematical Finance and empirical facts). The last two books and Musiela and Rutkowski (1997), which reviews hundreds of papers and results, focus mainly on martingale methods, whereas Wilmott *et al.* (1993, 1995) and Kwok (1998) use the PDE-approach and discuss relevant numerical methods. The monograph Dixit and Pindyck (1996) is an excellent exposition of the Gaussian Real Options theory for Economists, in the PDE-framework (mainly, 1D-case).

Non-Gaussian models (stable Lévy processes) have been introduced to Finance by Mandelbrot (1963); see also Fama (1965) and the collection of papers Mandelbrot (1997). On truncated Lévy distributions and their application to empirical studies of Financial markets, see Mantegna and Stanley (2000) and Bouchaud and Potters (2000). The last book contains theoretical results on non-Gaussian pricing and hedging of European options (under processes of Koponen's family as well), futures and forwards, and heuristic approximate methods for portfolio optimization and some other problems.

For stochastic volatility models see Zhu (2000), Barndorff-Nielsen and Shephard (2001c, 2001c, 2002), review paper Barndorff-Nielsen *et al.* (2001) and the bibliography there. Notice that there is an overlap of classes of SV-nodels and Lévy models; see the discussion in Barndorff-Nielsen and Shephard (2001a, 2001b, 2002) and Barndorff-Nielsen *et al.* (2001). It is remarkable that in op. cit. (see also the bibliography there), the volatility is driven by a Lévy process of Ornstein-Uhlenbeck type, and nevertheless, in some cases explicit pricing formulas for European options have been obtained (which is not typical for SV-models even in the Gaussian case; see however Heston (1993) and Duffie *et al.* (2000)). SV-model based on the hyperbolic motion, was constructed in Eberlein *et al.* (2001). For modelling of the evolution of the volatility surface, see Cont and da Fonséca (2001).