Chapter 1

A Brief History of Quantum Tunneling

Three years after the discovery of natural radioactivity in 1896, Elster and Geitel [1] found the exponential decay rate of radioactive substances experimentally. In 1900 Rutherford [2] introduced the idea of half-life of these chemicals, i.e. the time that the number of radioactive nuclei reach one-half of their original number. In 1905 Schweidler [3] showed the statistical nature of the decay. This means that the probability of disintegration of a nucleus does not depend on the time of its formation and also the time that a particular nucleus decays can only be predicted statistically. This idea was verified empirically by Kohlrausch [4] in 1906. Later experiments showed that the decay width Γ (which is related to the half-life τ by $\tau = \frac{\ln 2}{\Gamma}$) does not depend on external variables such as pressure, temperature or chemical environment.

The exponential law of decay can be written either in differential form as

$$\frac{dN(t)}{dt} = -\Gamma N(t), \tag{1.1}$$

or as an integral of Eq. (1.1), i.e.

$$N(t) = N_0 \exp(-\Gamma t), \tag{1.2}$$

where N_0 is the original number of nuclei (at $t = 0$), $N(t)$ is their number at $t > 0$, and Γ is the decay probability per unit time. For the rate of decay one can use either $T = \frac{1}{\Gamma}$ or the half-life $\tau = T \ln 2$. It should be pointed out that $N(t)$ is not the result of a single measurement but it is the average over a group of measurements, therefore $P(t) = \frac{N(t)}{N_0}$ is the probability that certain nucleus has not decayed at the time t ($t > 0$) and has remained in its initial state. Instead of $N(t)$ we can use $P(t) = \frac{N(t)}{N_0} = e^{-\Gamma t}$ which is usually referred to as the law of exponential decay.

The theory of α-radioactivity on the basis of quantum tunneling was proposed by Gamow [5] [6] [7] who found the well-known Gamow formula. The story of this discovery is told by Rosenfeld [8] who was one of the leading nuclear physicist of the twentieth century. " In my experience nuclear physic starts with the sudden appearance, one morning in the library of the Göttingen Institute, of a fair-haired giant, with shortsighted, half-shut eyes behind his spectacles, who introduced himself, with a broad smile, by declaring " I am Gamow." This pronouncement, at that time, could not provoke very much excitement. As it turned out that Professor Born would not be in for some time, I proposed to Gamow to go out for a walk. It was during the walk that he told me what he was doing.

He wanted to understand alpha radioactivity. Now, this seemed to me- and I think most physicists then would have had the same reaction - quite fantastic idea. All we knew about nuclei was that they were very small and that they had spin; this had just emerged from Pauli's interpretation of the hyperfine structure which spectroscopists had detected in the spectra of the heaviest atoms".

Gamow's first attempt was a failure. In that he assumed that the α particle is a point particle located in the Coulomb field of the nucleus. He found a continuous spectrum for its emission, and this was in contradiction with the empirical fact that that there are certain characteristic energies with which the particles are emitted. Later Gamow thought of combining the attractive nuclear forces with the Coulomb repulsion and this combination provided an effective barrier for the α particle (Chapter 25). He solved the Schrödinger equation with this effective potential and he imposed the "outgoing" wave boundary condition for large distances from the center of the nucleus. Gamow found that this two-point boundary condition problem (for $r = 0$, ψ must be finite and for $r \to \infty$, $\psi \to \frac{\exp(ikr)}{kr}$, (Chapter 5)) does not have a solution for the real energies, but for complex energies there are solutions. He interpreted the complex part of the energy as the decay width $\frac{\Gamma}{2}$, (or decay constant) of the disintegration and in this way he found the Geiger-Nuttall formula [9] which is a relation between $\frac{\Gamma}{2}$ and the energy of the emitted α-particle (Chapter 25) [5] [6] [7]. This work was completed shortly after Gamow's arrival at Göttingen. When in a weekly seminar at the Institute he presented his result it attracted much attention. Max Born was among the audience and he realized the significance of the theory. Born noticed that this idea is not only applicable to nuclear physics, but it is a general feature that must be present in other physical systems. He noticed that the cold emission of electrons from a metallic surface (Chapter 23) can be another example of this phenomena. Born being one of the founders of modern quantum mechanics criticized the foundation of Gamow's work arguing that the Hamiltonian is a Hermitian operator and its eigenvalues must be real, not complex, as Gamow had assumed. However the success of Gamow's result could not have been ignored. Therefore Born worked on this problem for few weeks and obtained the same result by considering Hermitian operators and states with real eigenvalues [10]. For this Born assumed that inside the nucleus there are stationary and distinct states, and the Coulomb potential outside the nucleus has a continuous spectrum which overlaps with the discrete energies inside. Now one can consider these two sets of wave functions (inside and outside) as a complete set of states and expand the original wave packet in terms of these, to obtain essentially the same result as Gamow's [10] (see also [11]).

About the same time as Gamow published his work, Gurney and Condon also submitted an article to the periodical Nature about α-decay [12]. Years later (1969), E.U. Condon recalled the history of the theory of tunneling [13] which we will briefly mention here. In 1928 Condon was hired as an assistant professor at Princeton University. There he met R.W. Gurney, a former student of E. Rutherford. At that time there were two published papers, one by Oppenheimer [14] and the other by Fowler and Nordheim [15]. These scientists had observed certain interesting and unusual features in the quantum mechanics of one-dimensional systems, and they had applied this new mechanics to understand the physics of the cold emission of the electrons. By reading these articles Gurney thought of applying the same idea to solve the problem of α-decay. At first he asked the opinion of the physicist H.R. Robertson about this approach, but he received no encouragement. Later when Gurney discussed his idea with Condon, Condon realized the potential of this theory, and they decided to collaborate on this project. Very soon they observed that it is not essential to know the shape of the potential inside the nucleus, only one had to assume that the interior potential becomes zero at a distance equal to the nuclear radius. They also observed that they can use the semi-classical or WKB approximation (Chapter 3) to calculate the wave function under the barrier. In this way Gurney and Condon found the solution to the Schrödinger equation for the radial wave function with the condition that the amplitude of the wave function must be large inside and small outside the nucleus. From the solution of the wave equation they found the decay width and the energy of the emitted α-particle approximately. Within few days this work was submitted to the periodical Nature (July 1928) [12], and later they published a detailed account and sent it to the Physical Review, and this was published in February of 1929 [16].

After submitting this work for publication, Condon and Gurney thought of applying the result of their work to the question of artificial disintegration of the atomic nucleus. They realized that in quantum mechanics the penetration inside the nucleus is possible with the low energy protons or α-particles, whereas according to the laws of classical mechanics for penetration the energy of these particles must be higher than the maximum height of the barrier. But they regarded this as an obvious conclusion of their work and did not bother to publish it. On the other hand Gamow in 1928 and 1929 published papers pointing out this implication of quantum tunneling. About the same time Gurney was thinking about resonant tunneling i.e. how a particle having a low energy equal to one of the quasistationary energies of the nucleus can easily penetrate the barrier. This work of Gurney was published in Nature (1929), [17] and according to Condon it deserves more attention than what it has received so far.

In 1930's and 1940's there were many attempts to relate the dynamics of the electron current in a system of metal-semiconductor which was used in rectifying the current, to the tunneling of electrons in solids. But the models were not realistic enough and usually quantum theory was predicting a current in the opposite direction of the observed current. With the discovery of transistors in 1947, the tunneling of electrons received renewed attention. In 1950 the construction of semiconductors like Ge and Si had advanced to a point were it was possible to manufacture semiconductors of given characteristics.

In 1957 L. Esaki discovered tunnel diode and this discovery proved the electron tunneling in solids conclusively [18]. Three years later i.e. in 1960, I. Giaever observed that if one or both of the metals are superconducting then the voltage-current curve provides interesting information regarding the state of superconductor(s). This experiment of Giaever was sufficiently accurate that it enabled one to measure the energy gap in superconductors. This gap appears when electrons form Cooper pairs, and the gap plays an essential role in the BCS theory of superconductivity [19].

The other major discovery was the theoretical work of B.D. Josephson in 1962 in connection with the tunneling between two superconductors separated by a thin layer of insulating oxide which serves as the barrier. Taking all of this as a single system, Josephson was able to predict the existence of a second current, i.e. the supercurrent in addition to the current found by Giaever, and this he showed is due to the tunneling of electrons in pairs [20].

Only very recently the tunneling of an individual atom, e.g. hydrogen on a metal surface such as copper has been observed directly. A remarkable (non-classical) feature of the experiment is that the tunneling rate increases as the surface gets colder [21] [22].

For a brief history of time in quantum tunneling see the papers of Steiberg [23] [24].