

## Preface to the first edition

Conscious of the frustration we experienced as we tried to learn group theory, and apply it to problems in physics, Fan Wang, Mei Juan Gao and I endeavored in 1974 to carry out a systematic reform of standard, or traditional group representation theory. Our aim was to establish a new approach to group representations in accordance with the concept and method used in quantum mechanics, so that it would be much more accessible to physicists and chemists.

The breakthrough came in late 1974. A new approach to group representation gradually came into being and with it a new method, the eigenfunction method (EFM), emerged for finding primitive characters, irreducible bases, Clebsch–Gordan (CG) coefficients, etc. From 1976–1978, the EFM was translated into codes and for the first time the CG coefficients and the outer-product reduction coefficients for the permutation group  $S_2$ – $S_6$  were computed by the EFM. The EFM was extended in 1979 to the computation of the CG coefficients, Racah coefficients, isoscalar factors, subduction coefficients of unitary groups. From 1981–1987, the new approach was applied to space groups and graded unitary groups and established several codes for space groups,  $SU(n)$  CG coefficients,  $SU(mn) \supset SU(m) \times SU(n)$  and  $SU(m+n) \supset SU(m) \times SU(n)$  coefficients of fractional parentage, and so on.

The new approach has been introduced at various universities and institutions, such as the City College of the City University of New York, the University of Chicago, Clarkson, Dalhousie, Drexel, Duke, Hong Kong, Minnesota, Maryland, Montreal, Singapore, Yale, and Los Alamos National Laboratory, as well as at the IXth and XIVth International Colloquium on Group Theoretical Method in Physics held in Mexico 1980, and Seoul 1985, respectively.

The Chinese book *A New Approach to Group Representation Theory* was written in 1981 and published in 1985 and has since been used as a textbook in Nanjing University for undergraduate and graduate students. Prior to the publication of the Chinese book, the manuscript had been used as a textbook in Nanjing University and Drexel University in Philadelphia.

The present book was based on the Chinese book and the new developments during the period of 1981–1987. All chapters except chapters 4 and 7 have been re-written.

I am indebted to my collaborators Mei–Juan Gao who took part in the writing of Chapter 8 and performed most of the practical computations, Guang–Qun Ma who was involved in the writing of Chapter 10, and it is he who first attempted to use the EFM to treat the space group representation, Fan Wang and Xuan–Gen Chen for their invaluable suggestions and criticism. I am grateful to Professors Xiao–Qian Zhou, Shi–Shu Wu and Duan Feng for their constant encouragement and patient reading of the manuscript. Thanks are due also to Professors Tan Lu, Fu–Cho Pu, Qin–Yue Qu, Hong–Zhou Sun, Rui–Bao Tao, Cheng–Li Wu, Rong–Jue Wei, Xi–De Xie and Guang–Xiang Xu for their enthusiasm in this work and support. I am grateful to many students who have offered helpful criticism and raised challenging questions.

Mr. Pei–Ning Wang typed the initial draft of Chapters 4–9, and my son, Bing–Qing wrote several software codes for facilitating the typing and proof-reading of the manuscript. Without their help it would be inconceivable that the re-writing could have been accomplished within

half a year.

Special mention must be made of Professors B. Bayman, M. Hamermesh and K.T. Hecht for their constant interest and many illuminating discussions, and D.H. Feng, who gave me the opportunity and encouragement to teach a course on group theory at Drexel University.

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## Preface to the second edition

Ten years have passed since the publication of the first edition of this book in which a complete set of commuting operators (CSCO) approach to the group representation theory was proposed. During this period, several remarkable advances of relevance have been made. A common feature of these new developments is that they give algebraic expressions for some useful coefficients of the following groups:

1. Quantum groups  $U_q(n)$  (Pan & Chen 1993).
2. Orthogonal groups  $O_n$  and symplectic groups  $Sp(2n)$  (Pan & Dai, 1996, Pan, Dong & Draayer, 1997, 1998).
3. Unitary groups and permutation groups with irreducible representations involving two columns (Li & Paldus, 1990, 1993).
4. Point groups (Chen & Fan 1998, Fan & Chen, 1999, Fan, Chen & Draayer 1999).

The first three will not be covered in this book but will be mentioned in appropriate places in the book, while some new results for point groups will be introduced in Chapter 8.

In the 1989 edition of the book, the projection operator method, the most commonly used method in standard textbooks on group theory, was essentially ignored. We mentioned only the main steps and discussed its serious shortcomings. Instead we developed the eigenfunction method (EFM) and urged its use. Ten years later, the wheel has gone full circle, and it is time to re-assess the power of the projection operator method. With the discovery of algebraic expressions for the projection operators, the projection operator method becomes extremely simple and powerful for point groups. An interesting point though is that only with the help of the EFM are these algebraic expressions obtainable. So we have a good example of negating a negation. The projection operator returns, albeit in a new form.

In this edition the following sections are new:

Sec. 3.14. Irreducible basis vectors in non-orthogonal reducible basis.

Sec. 5.9. The groups  $U_{2j+1}$ ,  $SO_{2l+1}$  and  $Sp_{2j+1}$ .

Sec. 7.20. The generalized quantized expressions for the coefficients of fractional parentage.

Sec. 9.8. The Groups  $Sp_N$ ,  $SO_N$  and the pairing interaction.

Sec. 9.2. Nuclear shell model: Multi-shell.

Most parts of Chapter 8 on the point groups have been rewritten. The new contents include a preliminary introduction of the algebraic solutions and a detailed discussion of the double-valued representations of point groups.

The assistance of Dr. L. McAven and Prof. C. W. Wong has been invaluable in preparing this edition, and I am very grateful to them both. Dr. McAven helped with the drawing of all of the figures, checked the text and formulas, and assisted with layout issues. Prof. C. W. Wong carefully read the book, in particular improving upon the exposition of Sec. 3.9.3. Discussions with Drs. Peng-Dong Fan, S. E. Bernades and Hong-Zhou Sun have also been useful.

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