

Contents

Foreword to the first edition	v
Preface to the first edition	vii
Preface to the second edition	ix
Addendum	xi
Contents	xiii
Glossary	xxiii
Introduction	1
Chapter 1 Elements of Group Theory	
1.1. The Definition of a Group	4
1.2. The Permutation Group S_n	7
1.2.1. The definition of S_n	7
1.2.2. Permutations expressed in terms of cycles and transpositions	8
1.3. Subgroups	9
1.4. Isomorphism and Homomorphism	10
1.5. Conjugate Classes	12
1.6. Cosets and Lagrange's Theorem	14
1.6.1. Left and right cosets	14
1.6.2. Lagrange's Theorem	15
1.6.3. Double cosets	15
1.7. Invariant Subgroups	16
1.8. Factor Groups*	17
1.9. Direct Product and Semi-Direct Product Groups	17
Chapter 2 Group Representation Theory	
2.1. Linear Vector Spaces	19
2.1.1. Defining linear vector spaces	19
2.1.2. Covariant and contravariant	20
2.1.3. The metric tensor	21
2.2. Linear Operators and their Representations	22
2.3. Complete Sets of Commuting Operators	24
2.3.1. The eigenspace of self-adjoint operators	24
2.3.2. A complete set of commuting operators (CSCO)	25

2.4.	Group Representations	26
2.5.	Unitary Representation	29
2.6.	Regular Representation and the Group Algebra	31
2.6.1.	Definition of the regular representation	31
2.6.2.	The group space	32
2.6.3.	The group algebra	32
2.7.	The Space of Functions on the Group	33
2.8.	Equivalent Representations and Characters	34
2.9.	Reducible and Irreducible Representations	35
2.10.	Subduced and Induced Representations	38
2.11.	Schur's Lemma	39
2.12.	Appendix: Non-Orthogonal Bases	40
2.12.1.	Two definitions of the representation of an operator	40
2.12.2.	Representations of adjoint operators	42
2.12.3.	Representations of unitary operators	42
2.12.4.	Representations under basis transformations	42
2.12.5.	Eigenvectors of a self-adjoint operator	43
Chapter 3	Representation Theory for Finite Groups	
3.1.	The Class Space and Class Operators	46
3.1.1.	Class Operators	46
3.1.2.	Class Algebra	48
3.1.3.	The Space of Functions on Classes	48
3.1.4.	The Natural Representation of a Class Algebra	49
3.2.	The First Kind of CSCO of G (CSCO-I)	51
3.2.1.	Reduction of the natural representation of the class algebra	51
3.2.2.	The CSCO-I of G	52
3.2.3.	The CSCO of a direct product group $G_1 \times G_2$	54
3.2.4.	The case of non-self-adjoint class operators	54
3.2.5.	The groups S_3 and C_{6v}	55
3.3.	The Projection Operator $P^{(\nu)}$	57
3.3.1.	Decomposition of the regular rep into inequivalent reps of G	57
3.3.2.	Labels for irreps	59
3.3.3.	Decomposition of an arbitrary rep space	60
3.4.	The Reduction of the Representations of C_{3v} , S_2 and S_3	62
3.4.1.	The group C_{3v}	62
3.4.2.	The group S_2	63
3.4.3.	The group S_3 in the configuration $\alpha^2\beta$	63
3.4.4.	The group S_3 in the configuration $\alpha\beta\gamma$	66
3.5.	The State Permutation Group	67
3.6.	Reduction of the Regular Rep of S_3	69
3.7.	The Intrinsic Group	71
3.7.1.	Definition of the intrinsic group	72
3.7.2.	The intrinsic state (regular rep case)	73
3.7.3.	The regular representation of intrinsic group	74
3.7.4.	Action of intrinsic group elements on functions on the group	75
3.7.5.	Properties of the intrinsic group	75
3.7.6.	Some remarks	76
3.7.7.	Intrinsic permutation group and state permutation group	76
3.8.	CSCO-II and CSCO-III of G	77
3.9.	Full Reduction of the Regular Representation	79
3.9.1.	Eigenvectors of the CSCO-III of G	79

3.9.2.	The representations $D^{(\nu)k}(G)$ and $D^{(\nu)m}(\overline{G})$	82
3.9.3.	The standard phase choice for $\mathbf{u}_{\nu mk}$	83
3.9.4.	The irreducibility of $D^{(\nu)}(G)$	84
3.9.5.	The EFM for $G \supset G(s)$ irreducible matrices	85
3.9.6.	Reduction of the regular representation in configuration space	86
3.9.7.	Example: the group S_3	87
3.10.	The Projection Operator $P_{mk}^{(\nu)}$ and the Generalized Projection Operator $\mathcal{P}_{mk}^{(\nu)}$	89
3.10.1.	Properties of the $P_{mk}^{(\nu)}$	89
3.10.2.	A recursive method for obtaining the $P_{mk}^{(\nu)}$	91
3.10.3.	Generalized irreducible matrices and generalized projection operator	93
3.10.4.	Coset factored projection operator	93
3.11.	The Eigenfunction Method for Characters	94
3.12.	Applications of Simple Characters	95
3.13.	Reduction of Non-Regular Reps (EFM for Irreducible Bases)	96
3.13.1.	Multiplicity free case ($\tau_\nu = 1$)	96
3.13.2.	Canonical subgroup chains with $\tau_\nu > 1$	97
3.13.3.	Non-canonical subgroup chains	100
3.13.4.	The projection operator method	100
3.14.	Irreducible Basis Vectors in a Non-Orthogonal Reducible Basis	101
3.15.	Kronecker Product of Representations	102
3.15.1.	Clebsch-Gordan series	102
3.15.2.	Symmetrized and anti-symmetrized squares	103
3.16.	The Clebsch-Gordan (CG) Coefficients	103
3.16.1.	Definition and properties of the CG coefficients	103
3.16.2.	The EFM for CG coefficients	105
3.17.	Isoscalar Factors	106
3.18.	Irreducible Tensors for a Group G	107
3.18.1.	The definition of an irreducible tensor	107
3.18.2.	Two kinds of invariants	108
3.18.3.	The Wigner-Eckart theorem	109
3.19.	Symmetries of the CG Coefficients and Isoscalar Factors	110
3.20.	Applications of Group Theory in Quantum Mechanics	112
3.20.1.	When G is the symmetry group of the Hamiltonian	112
3.20.2.	Splitting of the energy level due to a perturbation	114
3.20.3.	Dynamical symmetry	115
3.20.4.	The general case	115
3.20.5.	Selection rules	115
3.21.	Summary	116

Chapter 4 Representation Theory of the Permutation Group

4.1.	Partitions, Young Diagrams and Eigenvalues of CSCO-I	119
4.2.	Characters of Permutation Group	120
4.3.	Branching Laws, the Young-Yamanouchi Basis and Young Tableaux	121
4.4.	Yamanouchi Matrix Elements	122
4.5.	The CSCO-II of Permutation Groups	127
4.6.	The EFM for the Yamanouchi Basis (I)	132
4.7.	The CSCO-III of the Permutation Group	138
4.7.1.	CSCO-III	138
4.7.2.	The labeling for Yamanouchi basis of S_n and \overline{S}_n	139
4.7.3.	Phase convention and the principal term	140
4.7.4.	The matrix elements of conjugate irreps	141

- 4.7.5. The symmetrizer and anti-symmetrizer 141
- 4.8 The Quasi-Standard Basis of the Permutation Group 142
 - 4.8.1. The state permutation group (for the case with repeated state labels) . . . 142
 - 4.8.2. The quasi-standard basis of the permutation group 143
 - 4.8.3. Projection operator and quasi-standard basis 144
 - 4.8.4. The labelling of the quasi-standard basis 147
- 4.9 The EFM for the Yamanouchi Basis (II) 149
- 4.10. The Inner Product and the CG Series of Permutation Groups 152
- 4.11. Calculation of the CG Coefficients of Permutation Groups 153
- 4.12. Properties of the CG Coefficients of Permutation Groups 158
- 4.13. Tables of the CG Coefficients for S_3 — S_5 159
- 4.14. Outer-Product of the Permutation Group and the Littlewood Rule 165
- 4.15. The Calculation of the Induction Coefficients (IDC) of S_n 168
- 4.16. Properties of the IDC 170
- 4.17. Tables of the IDC for S_3 — S_5 173
- 4.18. The $S_{n_1+n_2} \supset S_{n_1} \otimes S_{n_2}$ Irreducible Basis 181
 - 4.18.1 The Frobenius reciprocity theorem and the $S_{n_1+n_2} \supset S_{n_1} \otimes S_{n_2}$ subduced basis 181
 - 4.18.2. Transformations between the standard basis and the non-standard bases of S_n 182
 - 4.18.3. The calculation of the subduction coefficients (SDC) 183
 - 4.18.4. Tables of the SDC for S_3 — S_6 187
- 4.19. The $S_n \supset S_{n_1} \otimes S_{n_2}$ Isoscalar Factors* 191
 - 4.19.1. The $S_n \supset S_{n-1}$ ISF 191
 - 4.19.2. Phase convention 194
 - 4.19.3. The properties of the $S_n \supset S_{n-1}$ ISF 194
 - 4.19.4. A special case 195
 - 4.19.5. Tables of the $S_n \supset S_{n-1}$ ISF 195
 - 4.19.6. The $S_{n_1+n_2} \supset S_{n_1} \otimes S_{n_2}$ ISF* 201
- 4.20. Appendix: Derivation of the Yamanouchi Matrix Elements by the EFM 202

Chapter 5 Lie Groups

- 5.1. Tensors 205
 - 5.1.1. Vectors (rank one tensors) 205
 - 5.1.2. Tensors with rank higher than one 206
 - 5.1.3. The metric tensor 206
 - 5.1.4. Metric spaces 207
- 5.2. Definition of a Lie Group; With Examples 208
- 5.3. Lie Algebras 210
- 5.4. Finite Transformations 213
- 5.5. Correspondence between Lie Groups and Lie Algebras 214
- 5.6. Linear Transformation Groups 216
- 5.7. Infinitesimal Operators for Linear Transformation Groups 218
- 5.8. The Metric Tensor in n -Dimensional Space and Infinitesimal Operators 221
 - 5.8.1. Unitary groups 222
 - 5.8.2. Infinitesimal operators of SU_n 223
 - 5.8.3. The group $U(n, m)$ 224
 - 5.8.4. The orthogonal group O_n 224
 - 5.8.5. The real orthogonal group $O(n, m)$ 225
 - 5.8.6. Symplectic groups 225
- 5.9. The Groups U_{2j+1} , SO_{2l+1} and Sp_{2j+1} 228
- 5.10. Infinitesimal Operators in Group Parameter Space 232

5.11. Isomorphism and Anti-Isomorphism of Lie Groups and Lie Algebras 233

5.12. Invariant Integration 235

5.13. Representations of Compact Lie Groups 236

 5.13.1. The fundamental representation 237

 5.13.2. Adjoint representations 237

 5.13.3. The metric tensor in the r -dimensional vector space 238

5.14. The Invariants and Casimir Operators of Lie Groups 239

5.15. Intrinsic Lie Groups 241

 5.15.1. Definition and interpretation of the intrinsic Lie group 241

 5.15.2. Infinitesimal operators of intrinsic groups in group parameter space 242

5.16. The CSCO Approach to the Rep Theory of Lie Groups 243

5.17. Irreducible Tensors of Lie Groups and Intrinsic Lie Groups 245

5.18. The Cartan–Weyl Basis 247

5.19. Theorems on Roots 251

5.20. Root Diagrams 252

5.21. The Dynkin Diagram and the Simple Root Representation 254

5.22. The Cartan Matrix 255

5.23. Theorems on Weights 256

5.24. The Dynkin Representation and the Chevalley Basis 259

 5.24.1. The Dynkin representation 259

 5.24.2. The eigenvalues of the Casimir operators 262

 5.24.3. The Chevalley basis 264

5.25. Algorithms for Computing the Roots and Weights 266

5.26. The Fundamental Weight System 272

5.27. The Fundamental Weight System Rep and the Cartesian Rep 273

5.28. Comparing the Different Representations 284

5.29. The Characters and CG Series of Lie Algebras 286

 5.29.1. The Characters of Lie Groups 286

 5.29.2. The CG Series of Lie Groups 287

Chapter 6 The Rotation Group

6.1. The Differential Operators $J_{x,y,z}$ and $\bar{J}_{x,y,z}$ in Group Parameter Space 289

6.2. Irreps of the Group SO_2 292

6.3. The CSCO–I and Characters of SO_3 293

6.4. The CSCO–III and Irreducible Matrix Elements of SO_3 297

6.5. The CSCO–II and Irreducible Bases of SO_3 299

6.6. The Intrinsic State of SO_3 300

6.7. The Projection State of SO_3 301

6.8. Irreducible Tensors of SO_3 and \overline{SO}_3 302

 6.8.1. The irreducible tensor $T_\rho^{(1)}$ of the adjoint rep of SO_3 and \overline{SO}_3 302

 6.8.2. Irreducible tensors $T_n^{(\nu)}$ of SO_3 and \overline{SO}_3 in general cases 303

Chapter 7 The Unitary Groups

7.1. Unitary Groups in Coordinate Space and State Space 306

7.2. Relations between Unitary and Permutation Groups 308

 7.2.1. The Gel’fand invariants 308

 7.2.2. The relation between CSCO–I’s of permutation and unitary groups 309

 7.2.3. Relations between the generators of unitary and permutation groups 312

7.3. The CSCO–II and CSCO–III of U_n and SU_n 312

7.4. The Gel’fand Basis and Gel’fand Matrix Elements 315

7.5.	The Gel'fand Basis of Unitary Groups and the Quasi-Standard Basis of Permutation Groups	316
7.5.1.	The CSCO-II of unitary groups and CSCO of the broken chains of permutation groups	316
7.5.2.	The labeling and finding of the Gel'fand basis	318
7.6.	The Contragredient Representation	326
7.7.	The CG Coefficients of SU_n Group	328
7.7.1.	The CG coefficients of U_n and the IDC of the permutation group	328
7.7.2.	The procedure for evaluating the SU_n CG coefficients	331
7.7.3.	Phase conventions	331
7.8.	The CG Coefficients of SU_n and the $S_f \supset S_{f_1} \otimes S_{f_2}$ Irreducible Basis	333
7.9.	The $SU_{mn} \supset SU_m \times SU_n$ Irreducible Basis	334
7.9.1.	The CG coefficients of S_f and the $SU_{mn} \supset SU_m \times SU_n$ irreducible basis	334
7.9.2.	The irreps ($[\nu_1], [\nu_2]$) of the groups SU_m and SU_n contained in the irrep $[\nu]$ of SU_{mn}	337
7.9.3.	Representation transformation between the $SU_{mn} \supset SU_m \times SU_n$ irreducible basis and the SU_{mn} Gel'fand basis	338
7.10.	The $SU_{n_1 n_2 n_3} \supset SU_{n_1} \times SU_{n_2} \times SU_{n_3}$ Irreducible Bases and the Racah Coefficients of Permutation Groups*	339
7.11.	The $SU_{n_1 n_2 n_3 n_4} \supset SU_{n_1} \times SU_{n_2} \times SU_{n_3} \times SU_{n_4}$ Irreducible Basis and the 9ν Coefficients of the Permutation Group*	341
7.12.	The $SU_{m+n} \supset SU_m \otimes SU_n$ Irreducible Basis	342
7.12.1.	The IDC of permutation groups and the $SU_{m+n} \supset SU_m \otimes SU_n$ irreducible basis	343
7.12.2.	The content of irreps ($[\nu_1], [\nu_2]$) of $SU_m \otimes SU_n$ in the irrep of SU_{m+n}	344
7.12.3.	The representation transformation between the irreducible basis $SU_{m+n} \supset SU_m \otimes SU_n$ and the Gel'fand basis of SU_{m+n}	344
7.13.	The Isoscalar Factors and the Fractional Parentage Coefficients	346
7.13.1.	Isoscalar factors	346
7.13.2.	The orbital fractional parentage coefficients (CFP)	348
7.13.3.	The spin-isospin CFP	350
7.13.4.	The total CFP	351
7.13.5.	The CFP for j - j coupling	352
7.13.6.	The eigenfunction method for evaluating the CFP	353
7.14.	The $S_f \supset S_{f_1} \otimes S_{f_2} \otimes S_{f_3}$ Irreducible Basis and SU_n Racah Coefficients*	358
7.15.	The $S_f \supset S_{f_1} \otimes S_{f_2} \otimes S_{f_3} \otimes S_{f_4}$ irreducible basis and the 9ν coefficients of SU_n *	360
7.15.1.	The 9ν coefficients of SU_n	360
7.15.2.	Evaluation of the Racah coefficients and 9ν coefficients of SU_n	362
7.16.	$SU_{mn} \supset SU_m \times SU_n$ CFP*	364
7.16.1.	$SU_{mn} \supset SU_m \times SU_n$ CFP and $S_{f_1+f_2} \supset S_{f_1} \otimes S_{f_2}$ ISF	364
7.16.2.	The evaluation of the $SU_{mn} \supset SU_m \times SU_n$ many-particle CFP	366
7.16.3.	Symmetries of the $SU_{mn} \supset SU_m \times SU_n$ ISF	368
7.16.4.	More examples	369
7.16.5.	$SU_{4(2l+1)} \supset (SU_{2l+1} \supset SO_3) \times (SU_4 \supset SU_2 \times SU_2)$ ISF and total CFP	371
7.17.	The $SU_{m+n} \supset SU_m \otimes SU_n$ CFP*	372
7.17.1.	The $S_f \supset S_{f-1}$ outer-product ISF (The $SU_f \supset SU_{f-1} \otimes U_1$ ISF)	372
7.17.2.	The $S_f \supset S_{f_{12}} \otimes S_{f_{34}}$ outer-product ISF ($SU_f \supset SU_{f_{12}} \otimes SU_{f_{34}}$ ISF)	376
7.17.3.	The $SU_{m+n} \supset SU_m \otimes SU_n$ ISF and $S_f \supset S_{f_{12}} \otimes S_{f_{34}}$ outer-product ISF	377
7.17.4.	The evaluation of $SU_{m+n} \supset SU_m \otimes SU_n$ ISF	378
7.17.5.	Symmetries of the $SU_{m+n} \supset SU_m \otimes SU_n$ ISF	378
7.18.	The SU_n Singlet Factor	379
7.19.	Second Quantized Expressions for the CFP	380

7.19.1.	One-particle CFP	380
7.19.2.	Two-particle CFP	383
7.19.3.	CFP in the interacting boson model	384
7.20.	Generalized Quantized Expressions for the CFP	385
7.20.1.	The generalized quantization	385
7.20.2.	The CFP in the generalized quantization	386
7.20.3.	The relation between the generalized and second quantization	387

Chapter 8 The Point Groups

8.1.	Basic Operations of Point Groups and Their Faithful Representations	389
8.2.	Some Commonly Used Point Groups	394
8.3.	Character Tables of Point Groups	400
8.3.1.	The conventional labelling for point group irreps (Mullikan notation)	400
8.3.2.	Character tables of point groups	400
8.4.	The CSCO-I and CSCO-II of Point Groups	400
8.4.1.	The CSCO-I of point groups	400
8.4.2.	The CSCO-II of point groups	401
8.4.3.	The codes for point groups	406
8.4.4.	The point group tables	406
8.5.	Algebraic Solutions for the Dihedral Groups D_n	407
8.5.1.	Factorization lemma for the projection operators	407
8.5.2.	The $D_n \supset C_{2x}$ generalized projection operators of D_n	409
8.5.3.	The $D_n \supset C_n$ generalized projection operators of D_n	410
8.5.4.	The $D_n \supset C_n$ projection operators	410
8.5.5.	The $D_n \supset C_{2x}$ projection operators	411
8.5.6.	The characters and irreducible matrices of D_n	411
8.5.7.	The symmetry adapted functions	412
8.5.8.	The group D_∞	415
8.5.9.	Improper dihedral groups C_{nv} , D_{nd} (even n) and D_{nh} (odd n)	416
8.6.	Numerical solutions for $T \supset D_2$ and $O \supset D_4 \supset D_2$	417
8.7.	Algebraic Solutions for Cubic Groups	420
8.7.1.	Double-coset factored projection operators and its application	420
8.7.2.	Algebraic expressions for the $T \supset C_3$ projection operator	421
8.7.3.	The $T \supset C_3$ irreducible matrices	424
8.7.4.	The algebraic expressions for $T \supset C_3$ SAF's	424
8.7.5.	SAFs for the group chain $O \supset T \supset C_3$	426
8.7.6.	The splitting of atomic levels in the $O_3 \supset G \supset G(s)$ basis	428
8.8.	The CG Coefficients of Point Groups	429
8.8.1.	The CG series of point groups	429
8.8.2.	The CG coefficients of point groups	429
8.9.	Molecular Orbital Theory	431
8.10.	Single Electron SALC	433
8.11.	Double Point Groups for d-v Representations	441
8.11.1.	The double point group method	441
8.11.2.	Euler angles and group tables	442
8.11.3.	Some basic relations between point group operators	443
8.11.4.	The Opechowski rule for classes	445
8.11.5.	The double-group method for d-v representations	446
8.12.	The Representation Group and Its Applications	447
8.12.1.	The representation group	447
8.12.2.	Characters of d-v irreps of point groups	451
8.12.3.	Algebraic solutions for cyclic groups C_n , S_{2n} and C_{nh}	452

8.12.4.	Algebraic solutions for dihedral groups D_n in d-v reps	456
8.13.	The Time Reversal Symmetry	457
8.13.1.	The time reversal operator	457
8.13.2.	The time reversal group	458
8.13.3.	Three types of irreps	460
8.13.4.	Degeneracy due to time reversal symmetry	460
8.13.5.	The transformation of irreducible basis under time reversal	461
Chapter 9 Applications of Group Theory to Many-Body Systems*		
9.1.	Nuclear Shell Model: Single-Shell	464
9.1.1.	First quantization	465
9.1.2.	Generalized quantization	465
9.2.	Nuclear Shell Model: Multi-Shell	468
9.3.	Anti-Symmetric Wave Functions for an A+B System	470
9.4.	Transformations between Symmetry Bases and Physical Bases in the Quark Model	473
9.5.	The Dynamical Symmetry Models of Nuclei	474
9.6.	The Quasispin Model	475
9.7.	The Proton-Neutron Quasispin Model	477
9.8.	The Groups Sp_N, SO_N and the Pairing Interaction	480
9.8.1.	Pairing interaction for identical particles	480
9.8.2.	Pairing interaction for electrons and non-identical nucleons	481
9.9.	The Elliott Model	483
9.10.	The Interacting Boson Model*	487
9.11.	The Molecular Shell Model	492
9.11.1.	The Hamiltonian as a function of infinitesimal operators of the unitary group	493
9.11.2.	Spin-free approximation	493
Chapter 10 The Space Groups		
10.1.	The Euclidean Group	495
10.1.1.	Definition of the Euclidean Group	495
10.1.2.	Properties of the Euclidean Group Operators	496
10.2.	The Lattice Group	497
10.3.	The Space Group	498
10.4.	The Point Group \mathbf{P} and the Crystal System	499
10.5.	The Bravais Lattice	500
10.6.	Operators of the Space Group	502
10.6.1.	The properties of group operators	502
10.6.2.	Example: Group D_{4h}^{14}	503
10.7.	The Reciprocal Lattice Vectors	504
10.8.	Irreps of the Lattice Group	505
10.9.	The Brillouin Zone	507
10.10.	The Electron State in a Periodic Potential	508
10.11.	Representation Space of the Space Group	508
10.12.	The Little Group $\mathbf{G}(\mathbf{k})$	509
10.13.	The Representation Groups $G_{\mathbf{k}}$ and $G'_{\mathbf{k}}$	510
10.13.1.	The rep group $G_{\mathbf{k}}$	510
10.13.2.	The rep group $G'_{\mathbf{k}}$	512
10.13.3.	Special cases of the rep group $G'_{\mathbf{k}}$	513
10.14.	The Irreducible Basis and Matrices of $G'_{\mathbf{k}}$	514
10.14.1.	The group table of $G'_{\mathbf{k}}$	514

10.14.2. The CSCO-II and CSCO-III of $\mathbf{G}'_{\mathbf{k}}$ 515

10.14.3. The irreps of $\mathbf{G}'_{\mathbf{k}}$ and the projective irreps of $\mathbf{G}_0(\mathbf{k})$ 516

10.14.4. The irreducible basis of $\mathbf{G}(k)$ 517

10.15. Example: the Point W of O_h^7 517

10.15.1. Seeking the CSCO and the characters of the point W of the space group O_h^7 517

10.15.2. Obtaining the CSCO-I from the existing character table 519

10.15.3. Constructing irreps of the rep group $\mathbf{G}'_{\mathbf{k}}$. The point W of O_h^7 520

10.16. Irreducible Basis and Representations of the Space Group 520

10.16.1. The \mathbf{k} star 520

10.16.2. The induced rep 523

10.16.3. A simple algorithm for full rep matrices 524

10.16.4. The $\mathbf{G} \supset \mathbf{G}(\mathbf{k}_\sigma) \supset \mathbf{G}(s_\sigma) \supset \mathbf{T}$ irreducible basis 526

10.17. The Irreducible Basis and Matrices of C_{2v}^4 527

10.17.1. A general star: $\mathbf{p} = (p_1, p_2, p_3)$ 527

10.17.2. The star $\Gamma : \mathbf{p} = (0, 0, 0)$ 528

10.17.3. The star $\Sigma : \mathbf{p} = (p_1, 0, 0)$ 529

10.17.4. The star $X : \mathbf{p} = (\frac{1}{2}, 0, 0)$ 530

10.18. The Clebsch-Gordan Coefficients of Space Groups* 530

10.18.1. The CG series 530

10.18.2. The calculation of the CG coefficients 532

10.18.3. Relative phase of the CG coefficients 533

10.18.4. The full CG coefficients of space groups 534

10.18.5. A summary of the eigenfunction method for space group CG coefficients 534

10.19. Examples: Getting Space Group Clebsch-Gordan Coefficients* 536

10.19.1. The CG coefficients of O_h^7 for $*X(1) \otimes *X(2) \rightarrow *X(\nu'')$ 536

10.19.2. The CG coefficients of O_h^7 for $*X(1) \otimes *W(1) \rightarrow *\Delta(\nu'')$ 540

10.20. The Double Space Groups 542

10.21. Appendix 546

Appendix

Table A1. Dimensions of irreps of the permutation group $S_f (f \leq 6)$ and the unitary groups $SU_n (n \leq 6)$ 549

Table A2. Phase factors $\varepsilon_1(\nu_1 \nu_2 \nu)$ for the permutation group IDC and SU_n CG coefficients 549

References 551

Index 567

Contents of Some Important Tables

3.2-1. The new and old labelling schemes for irreps of permutation groups	53
3.9. The standard basis $\psi_m^{(\nu)k} (P_m^{(\nu)k})$ of S_3 and \bar{S}_3 and the standard matrix elements	88
4.4-1. The phase factor $\Lambda_m^{[\nu]}$, Young tableau $Y_m^{[\nu]}$ and the corresponding eigenvalues $\lambda = \sum_{f=3}^n (2f-5)\lambda_f$	123
4.4-2. Yamanouchi matrix elements of adjacent transpositions for S_2 – S_5	125
4.8. Normalization factors $R^{(\nu)k}(\omega)$	146
4.10. CG series of S_3 – S_5	153
4.13. Tables of the CG coefficients of the permutation groups S_3 – S_5	160
4.14-2. Outer-product reduction rule	168
4.17. The $([\nu_1] \otimes [\nu_2]) \uparrow [\nu]$ IDC of the permutation groups	174
4.18. The SDC for S_3 – S_6	187
4.19. Tables of $S_n \supset S_{n-1}$ ISF for $n = 3$ – 5 (that is tables of the $SU_{mn} \supset SU_m \times SU_n$ single particle CFP for arbitrary m and n)	196
10.21-1. The group table of O^\dagger	546
10.21-2. The operations of point group elements on primitive translations \mathbf{t}_i	547