

Preface

The text of this book has its origin in a course we delivered to students for Master Degree at the Faculty of Mathematics of the University “Al. I. Cuza” Iași, Romania.

One can ask if another book on Convex Analysis is needed when there are many excellent books dedicated to this discipline like those written by R.T. Rockafellar (1970), J. Stoer and C. Witzgall (1970), J.-B. Hiriart-Urruty and C. Lemaréchal (1993), J.M. Borwein and A. Lewis (2000) for finite dimensional spaces and by P.-J. Laurent (1972), I. Ekeland and R. Temam (1974), R.T. Rockafellar (1974), A.D. Ioffe and V.M. Tikhomirov (1974), V. Barbu and Th. Precupanu (1978, 1986), J.R. Giles (1982), R.R. Phelps (1989, 1993), D. Azé (1997) for infinite dimensional spaces.

We think that such a book is necessary for taking into consideration new results concerning the validity of the formulas for conjugates and sub-differentials of convex functions constructed from other convex functions by operations which preserve convexity, results obtained in the last 10–15 years. Also, there are classes of convex functions like uniformly convex, uniformly smooth, well behaving, well conditioned functions that are not studied in other books. Characterizations of convex functions using other types of derivatives or subdifferentials than usual directional derivatives or Fenchel subdifferential are quite recent and deserve being included in a book. All these themes are treated in this book.

We have chosen for studying convex functions the framework of locally convex spaces and the most general conditions met in the literature; even when restricted to normed vector spaces many results are stated in more general conditions than the corresponding ones in other books. To make

this possible, in the first chapter we introduce several interiority and closedness conditions and state two strong open mapping theorems.

In the second chapter, besides the usual characterizations and properties of convex functions we study new classes of such functions: cs-convex, cs-closed, cs-complete, lcs-closed, ideally convex, bcs-complete and li-convex functions, respectively; note that the classes of li-convex and lcs-closed functions have very good stability properties. This will give the possibility to have a rich calculus for the conjugate and the subdifferential of convex functions under mild conditions. In obtaining these results we use the method of perturbation functions introduced by R.T. Rockafellar. The main tool is the fundamental duality formula which is stated under very general conditions by using open mapping theorems.

The framework of the third chapter is that of infinite dimensional normed vector spaces. Besides some classical results in convex analysis we give characterizations of convex functions using abstract subdifferentials and study differentiability of convex functions. Also, we introduce and study well-conditioned convex functions, uniformly convex and uniformly smooth convex functions and their applications to the study of the geometry of Banach spaces. In connection with well-conditioned functions we study the sets of weak sharp minima, well-behaved convex functions and global error bounds for convex inequality systems. The chapter ends with the study of monotone operators by using convex functions.

Every chapter ends with exercises and bibliographical notes; there are more than 80 exercises. The statements of the exercises are generally extracted from auxiliary results in recent articles, but some of them are known results that deserve being included in a textbook, but which do not fit very well our aims. The complete solutions of all exercises are given. The book ends with an index of terms and a list of symbols and notations.

Even if all the results with the exception of those in the first section are given with their complete proofs, for a successful reading of the book a good knowledge of topology and topological vector spaces is recommended.

Finally I would like to thank Prof. J.-P. Penot and Prof. A. Göpfert for reading the manuscript, for their remarks and encouragements.

C. Zălinescu
March 1, 2002
Iași, Romania