

## Preface

The bell curve, also known as the normal or Gaussian distribution, is one of the most ubiquitous mathematical objects in science. Introduced in the early nineteenth century by mathematical prodigy Karl Friederich Gauss, the curve's fundamental nature stems from the renowned central limit theorem, first proven by Pierre-Simon Laplace, that states that a bell is the universal curve found when a multitude of unrelated (independent) quantities are added together.

Recently, generalized versions of the central limit theorem leading to Gaussian distributions over one and higher dimensions, via arbitrary iterations of simple mappings, have been discovered by the author and his collaborators. The purpose of this book is to reveal how these new constructions lead to infinite exotic kaleidoscopic decompositions of two-dimensional circular bells in terms of beautiful deterministic patterns possessing arbitrary  $n$ -fold symmetries, patterns that, while reminding us of the infinite structure previously found in the celebrated Mandelbrot set, turn out to contain natural shapes such as snow crystals and biochemical rosettes, including life's own DNA.

This book is divided into three main sections. A general introduction to the ideas is given first, followed by a gallery of patterns found inside the bell when the iterations are guided by the binary expansion of the omnipresent number  $\pi$ , including a potpourri of images showing curious pattern evolutions and collages of bell patterns. Hoping to capture the most general readership, the introduction relies on several diagrams and uses as few equations as possible, leaving the technical details to a set of notes at the end of the book. This set of notes also points out relevant references, contains a sample program the reader may implement in his/her own computer in order to explore the many treasures found inside the bell, and includes the specific information for all the images in the book. To allow the reader to further appreciate the work, the book includes a CD containing selected bell patterns whose interesting evolutions may be readily visualized on a personal computer.

This book could not have been accomplished without the invaluable assistance of many people. First, my warmest thanks go to Michael F. Barnsley, whose lovely ideas shaped

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Finally, I would like to acknowledge the loving support of my beloved family and friends. By your constancy, you nurture and inspire my yielding to a majestic bell, one ever conducting.

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