Preface

This book is an introduction to the two-component spinor, or 2-spinor, formalism in general relativity. It has been designed to be readily accessible to those at graduate and research level.

The general mathematical framework of spinors was first introduced by the French mathematician Élie Cartan (1869–1951). Now some ninety years after its conception, spinors are used extensively in the fields of general relativity, quantum mechanics, particle physics, superstrings and M-Theory. Most theoretical physicists are familiar with 4-spinors due to the work of Dirac, and those involved in ten-dimensional superstring theory frequently employ 32-spinors. However, it is primarily through the work of Penrose that the 2-spinor formalism is widely used today in general relativity. Many of the notations and conventions introduced by Penrose are adhered to in this book; particularly those given in Penrose and Rindler, 1984, 1986. Part of the intention here is to equip the reader with the necessary information and techniques to enable him or her to progress to more detailed expositions on the subject, such as the material covered in the aforementioned text and appropriate research articles.

Chapter 1 begins by developing the 2-spinor formalism in a geometrical fashion. This is carried out by associating the real Minkowski coordinates on a Riemann sphere with a single complex one on a complex 2-plane via stereographical projections. A one-to-one correspondence can thereby be established between coordinate systems provided that each single complex 'point' is defined in terms of a complex 2-form. The elements of the 2-form will then be seen to form an orthonormal basis in spin-space. Each point in spin space can then be associated with a geometrical object called a spin-vector — the simplest kind of 2-spinor.

In Chapter 2 we build the spinor algebra and construct more general spinors. The connection between spinors and tensors is established through the use of Infeld–van der Waerden symbols. Furthermore, the decomposition of certain bivalent tensors into their spinor equivalents is discussed. Chapter 2 ends with the algebraic classification of the electromagnetic and Weyl spinors.

Chapter 3 commences with the axiomatic description of the spinor covariant derivative. We go on to introduce the spinor affine connections (Ricci rotation coefficients), curvature spinor and associated spinor fields. This culminates in a discussion of the Newman–Penrose spin coefficient formalism.

The main theme of Chapter 4 is the Lanczos spinor and the Weyl–Lanczos equations. The origins of this spinor are first highlighted together with its significance in general relativity. The Weyl–Lanczos equations are derived (using techniques given in Chapters 2 and 3) and some solutions are obtained.

For the benefit of advanced undergraduates who have done little or no general relativity, but nevertheless wish to gain some knowledge of the 2-spinor formalism, an appendix has been included to review the fundamentals of general relativity.

A number of selected exercises have been given at the end of each chapter.

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