

Contents

Preface	vii
Chapter 1 Descriptive Statistics	1
1.1. Introduction	1
1.2. Measures of Location	3
1.2.1. Mean	3
1.2.1.1. Some properties of mean	5
1.2.2. Median	6
1.2.3. Mode	7
1.3. Measures of Dispersion	8
1.3.1. Range	9
1.3.2. Variance and standard deviation	9
1.3.2.1. Some properties of variance and standard deviation	10
1.3.2.2. Two alternative formulas for the variance	11
1.3.3. Covariance $\text{Cov}(X, Y)$, or $S_{X,Y}$	11
1.4. Grouped Data	12
1.4.1. Computational formulas for grouped data	13
1.5. Graphics	15
1.5.1. Histogram	16

1.5.2.	Relative frequency polygon	16
1.5.3.	Cumulative frequency polygon	17
1.5.4.	Percentiles and quartiles and interquartile range	18
1.6.	An Example	20
1.7.	Proofs of the Results in this Chapter	22
1.8.	Exercises and Problems	23
Chapter 2 Probability		25
2.1.	Basic Concepts	25
2.1.1.	Three components of probability	25
2.1.2.	Definition of probability	26
2.1.3.	Values of a probability	29
2.1.4.	Sure event and impossible event	29
2.1.5.	Complement of an event (or negation of an event)	29
2.2.	Composite Event “ A and B ”	30
2.2.1.	Conditional probability	32
2.2.2.	Multiplication theorem	34
2.2.3.	Bayes’ theorem	35
2.2.4.	Independence of events	37
2.2.5.	A theorem of pairwise independence	38
2.2.6.	Multiplication theorem when events are independent	39
2.3.	Composite Event “ A or B ”	40
2.3.1.	Addition theorem	40
2.3.2.	Composite events of complements	41
2.3.3.	De Morgan’s laws	43
2.3.4.	Addition theorem when events are mutually exclusive	44
2.4.	Remarks on the Addition and Multiplication Theorems	44
2.4.1.	Summary of the two theorems	44
2.4.2.	The associative and distributive laws	44
2.5.	Factorials, Permutations, and Combinations	46
2.5.1.	Permutations	46
2.5.2.	Combinations	48
2.6.	Proofs of the Results in this Chapter	50
2.7.	Exercises and Problems	51
Chapter 3 Random Variables		55

3.1.	Definition of a Random Variable	55
3.1.1.	Illustrative examples	56
3.2.	Joint Probability Distribution	57
3.2.1.	Marginal probability	58
3.2.2.	Conditional probability	59
3.2.3.	Independence of random variables	59
3.3.	Expectation of a Random Variable	62
3.3.1.	Some properties of the expectation of a random variable	65
3.3.2.	Expectation of a linear function of random variables	66
3.3.3.	Expectation of a sample mean	67
3.4.	Variance of a Random Variable	67
3.4.1.	Two properties of the variance	69
3.4.2.	Variance of a linear function of independent random variables	70
3.4.3.	Variance of sample mean	70
3.4.4.	Expectation of sample variance	71
3.5.	Covariance of a Bivariate Distribution	71
3.5.1.	Some properties of the covariance	72
3.5.2.	The variance of a linear function of random variables	72
3.6.	Continuous Random Variables	74
3.6.1.	Joint probability distribution of two random variables	74
3.7.	Proofs of the Results in this Chapter	76
3.8.	Exercises and Problems	78
Chapter 4 Probability Distributions		81
4.1.	Introduction	81
4.2.	Uniform Distribution (Discrete)	82
4.3.	The Binomial Distribution, $B(n; p)$	83
4.4.	The Hypergeometric Distribution	91
4.4.1.	Relation with the binomial distribution	93
4.4.2.	Applications of the hypergeometric distribution	94
4.5.	The Poisson Distribution	96
4.6.	The Multinomial Distribution	99
4.6.1.	The covariance between X_i and X_j	100

4.7.	Continuous Uniform Distribution	103
4.8.	The Exponential Distribution	105
4.9.	The Normal Distribution	107
4.9.1.	Density function and distribution function	109
4.9.2.	The standard normal distribution $N(0, 1)$	111
4.10.	Proofs of the Results in this Chapter	114
4.11.	Exercises and Problems	121
Chapter 5 Statistical Inference—Interval Estimation		127
5.1.	Introduction	127
5.1.1.	The central limit theorem	129
5.1.2.	Normal approximation for the binomial distribution	133
5.2.	Interval Estimation of Population Means	136
5.2.1.	Interval estimation of a population mean when the standard deviation is known	136
5.2.1.1.	Length of the confidence interval	139
5.2.2.	Interval estimation of a population mean when the standard deviation is unknown	141
5.2.3.	Confidence interval for the difference between two population means when population variances are known	143
5.2.4.	Confidence interval of the difference between two population means when the population variances are unknown	144
5.3.	Confidence Intervals for Population Proportions	146
5.3.1.	Confidence intervals for a single population proportion, p	146
5.3.2.	Confidence interval for the difference between two population proportions	147
5.4.	Exercises and Problems	147
Chapter 6 Hypothesis Testing — Fundamental Concepts		151
6.1.	Introduction	151
6.2.	Basic Elements in Hypothesis Testing	153
6.2.1.	Statistical hypothesis	153
6.2.1.1.	Simple hypothesis versus composite hypothesis	154

6.2.2.	Statistical test and two hypotheses	154
6.2.3.	Two kinds of errors	156
6.2.4.	Level of significance and power of test	157
6.2.5.	Statistic (or test statistic) and critical ratio	158
6.2.6.	Critical region and region of acceptance	159
6.2.7.	Computation of power, $1 - \beta$	159
6.3.	Illustrative Examples of Hypothesis Testing	160
6.4.	Power of Test	168
6.4.1.	Determination of sample size n	169
6.5.	Power of Test Concerning a Single Probability	171
6.6.	Exercises and Problems	172
Chapter 7	Testing Hypotheses Concerning Population Means and Population Proportions	175
7.1.	Introduction	175
7.2.	The “Student’s” t Distribution	176
7.2.1.	One-sample case	176
7.2.2.	Two-sample case	178
7.3.	Procedure for Hypothesis Testing	180
7.4.	Test of Hypotheses Concerning Population Means	181
7.4.1.	Test of hypotheses concerning a single population mean	182
7.4.2.	Test of hypotheses concerning two population means	183
7.4.3.	Two populations — before and after treatment	184
7.4.4.	Some remarks regarding hypothesis testing	188
7.5.	Test of Hypotheses Concerning Probabilities	191
7.5.1.	Hypothesis testing concerning a single probability	192
7.5.2.	Hypothesis testing concerning two probabilities	193
7.6.	Exercises and Problems	196
Chapter 8	The Chi-Square Test	203
8.1.	Introduction	203
8.1.1.	The chi-square distribution	204
8.2.	Some Theorems Concerning the Chi-Square Distribution	205
8.3.	Chi-Square Tests	207
8.3.1.	One-way classification	207

8.3.1.1.	The chi-square test versus the normal Z test I	211
8.3.2.	Two-way classification	212
8.3.2.1.	Conditional distribution of Y given X	216
8.3.2.2.	The 2×2 contingency table	220
8.3.2.3.	The chi-square test versus the normal Z test II	221
8.3.2.4.	Case-control studies	223
8.3.2.5.	Relative risk and odds ratio	226
8.3.3.	Goodness of fit	227
8.4.	More Remarks about the Chi-Square Test	231
8.5.	Proof of the Results in this Chapter	233
8.6.	Exercises and Problems	235
Chapter 9 Linear Regression		243
9.1.	Introduction	243
9.2.	Estimation of a and b	245
9.3.	Underlying Assumptions	252
9.4.	Relevant Theorems	254
9.5.	Statistical Inference in Linear Regression	256
9.5.1.	Hypothesis testing regarding b	256
9.5.2.	Confidence interval for the expectation $E(Y X_0) = a + bX_0$	258
9.5.3.	Confidence belt for the regression line $E(Y X) = a + bX$	259
9.6.	Test of the Linear Regression Assumption	265
9.6.1.	The F distribution	266
9.6.2.	The F test of linear regression	267
9.6.3.	Test of hypothesis $b = 0$	268
9.7.	Remarks	272
9.7.1.	A linear model	272
9.7.2.	Choice of values of X	273
9.7.3.	Interval of prediction for an individual Y	273
9.7.4.	Interpolation versus extrapolation	274
9.8.	Some Other Forms of Regression	274
9.8.1.	Curve-linear regression	274
9.8.2.	Logistic regression	275
9.9.	Proofs of the Results in this Chapter	276

9.10. Exercises and Problems	278
Chapter 10 Correlation	281
10.1. The Correlation Coefficient, ρ_{XY} (or ρ)	281
10.1.1. The sample correlation coefficient, r_{XY}	283
10.2. Relationship Between Correlation and Regression	284
10.3. The Bivariate Normal Distribution	288
10.4. Statistical Inference About ρ_{XY}	291
10.4.1. Test of zero correlation	291
10.4.2. Confidence interval for ρ_{XY}	293
10.4.3. Spurious correlation?	296
10.5. Other Types of Correlation	300
10.5.1. Rank correlation coefficient (Spearman's correlation coefficient), r_s	300
10.5.2. Point biserial correlation coefficient, r_{pb}	302
10.5.3. The Φ coefficient	304
10.6. Proofs of the Results in this Chapter	306
10.7. Exercises and Problems	314
Chapter 11 Multiple Regression and Correlation	319
11.1. The Regression Equation	319
11.2. Estimation of a , b_1 , b_2	320
11.3. Theorems and Inferences	323
11.4. Multiple Correlation Coefficient, $\rho_{y \cdot 12}$	329
11.4.1. Test of hypothesis concerning true correlation coefficient $\rho_{y \cdot 12}$	330
11.5. Partial Correlation	332
11.6. A General Multiple Regression	335
11.6.1. The regression equation and estimation of the constants	335
11.6.2. Some theorems for statistical inference	338
11.6.3. Multiple correlation coefficient, $\rho_{y \cdot 12 \dots s}$	340
11.7. Multiple Logistic Regression	341
11.8. Proofs of the Results in this Chapter	342
11.9. Exercises and Problems	345
Chapter 12 One-Way Analysis of Variance	349
12.1. Introduction	349

12.2.	One-Way Analysis of Variance	350
12.2.1.	The layout and linear model	350
12.2.2.	Least-squares estimates, sums of squares (ssq), and mean squares (msq)	352
12.2.3.	Null hypothesis and test statistic	355
12.3.	Examples	358
12.4.	Multiple Comparisons	362
12.4.1.	The least significant difference (LSD)	362
12.4.2.	The studentized range	364
12.4.3.	Contrasts	365
12.4.3.1.	Bonferroni's inequality	366
12.4.4.	The t test	368
12.5.	Random-Effects Model	373
12.5.1.	Partition of sums of squares	375
12.5.2.	The null hypothesis	376
12.6.	A Comparison of Two Models	378
12.7.	Proofs of the Results in this Chapter	380
12.8.	Exercises and Problems	385
Chapter 13 Two-Way Analysis of Variance —		
	Fixed-Effects Models	389
13.1.	Introduction	389
13.2.	The Linear Model	391
13.3.	One Observation Per Cell	394
13.3.1.	Least squares estimates and sums of squares (ssq)	395
13.3.2.	Null hypotheses and test statistics	396
13.3.3.	Multiple comparisons	403
13.4.	n Observations Per Cell	404
13.4.1.	Linear model	406
13.4.2.	Least squares estimates and sums of squares (ssq)	407
13.4.3.	Null hypotheses and test statistics	408
13.4.4.	Multiple comparisons	414
13.5.	Proofs of Results in this Chapter	420
13.6.	Exercises and Problems	425
Chapter 14 Two-Way Analysis of Variance		
	Random-Effects Models and Mixed Model	429
14.1.	Introduction	429

14.2. One Observation Per Cell — Random-Effects Model . . .	430
14.2.1. Sums of squares and mean squares	431
14.2.2. Null hypotheses and test statistics	432
14.3. n Observations Per Cell — Random-Effects Model	434
14.3.1. Sums of squares and mean squares	435
14.3.2. Null hypotheses and test statistics	437
14.4. A Comparison of Two Models — Fixed-Effects Model versus Random-Effects Model	440
14.4.1. Expectations and variances	440
14.4.2. Statistical inference and test statistics	442
14.5. Mixed Model	446
14.5.1. Effects and interactions	447
14.5.2. Estimation of effects and sums of squares	448
14.5.3. Expectations of mean squares	449
14.5.4. Hypotheses and test statistics	451
14.6. Proofs of the Results in this Chapter	456
14.7. Exercises and Problems	464
 Chapter 15 Design of Experiment	 467
15.1. Introduction	467
15.1.1. Randomization	469
15.1.2. Replication	469
15.2. Completely Randomized Design	470
15.2.1. Table of random numbers	471
15.2.2. Method of analysis	472
15.3. Randomized Blocks — Single Blocking	473
15.3.1. Some remarks	473
15.3.2. An illustration	474
15.3.3. Linear model and F ratios	475
15.3.4. Efficiency of blocking	476
15.3.4.1. Measurement of efficiency	477
15.3.4.2. Efficiency expressed in F ratio	479
15.4. Latin Squares — Double Blocking	479
15.4.1. Examples	481
15.4.2. Symmetry of the three factors	483
15.4.3. Replication of Latin squares	484
15.4.4. Observations in a Latin square	486
15.4.5. Linear model and hypotheses	487

15.4.6. Sums of squares and test statistics	489
15.4.7. Analysis of data — an example	492
15.4.8. Efficiency of Latin square	496
15.4.8.1. Preliminaries	496
15.4.8.2. Efficiency relative to randomized blocks	498
15.4.8.3. Efficiency relative to complete randomization	499
15.5. Graeco-Latin Square — Triple Blocking	503
15.6. Proofs of the Results in this Chapter	507
15.7. Exercises and Problems	514
 Chapter 16 Analysis of Covariance	 519
16.1. Introduction	519
16.2. Linear Model, Hypotheses, and Regression Equations . . .	522
16.2.1. Null hypotheses	522
16.2.2. Regression equations and regression lines	524
16.3. Partition of Sum of Squares and F Statistics	530
16.3.1. The F statistics	532
16.3.1.1. Test of hypothesis $H_1 : E(Y_i X_i) = a_{iw} + b_w X_i$	532
16.3.1.2. Test of hypothesis $H_2 : E(Y_i X_i) = a_t + b_t X_i$	533
16.3.1.3. Test of hypothesis $H_3 : E(\bar{Y}_i \bar{X}_i) = a_b + b_b \bar{X}_i$	533
16.3.1.4. Test of hypothesis $H_4 : b_w = b_b$	535
16.3.1.5. Test of hypothesis $H_5 : b_w = 0$	536
16.4. Computation Formulas of the Sums of Squares	537
16.5. An Example — Oral Contraceptives and Metabolic Rate in Women	540
16.6. Proofs of the Results in this Chapter	544
16.7. Exercises and Problems	549
 Chapter 17 Non-Parametric Statistics	 555
17.1. Introduction	555
17.2. Sign Test	556
17.3. Wilcoxon Signed Rank Test	558
17.3.1. Critical values of R^+	558

17.3.2. Normal approximation	560
17.4. Wilcoxon Two-Sample Rank-Sum Test	560
17.4.1. Normal approximation	563
17.4.2. Test for equality of two variances	564
17.5. Mann–Whitney Test	565
17.6. Kruskal–Wallis Test — One-Way Analysis of Variance	566
17.7. Friedman Test — Two-Way Analysis of Variance	569
17.8. Proofs of the Results in this Chapter	574
17.9. Exercises and Problems	580
Appendix: Tables	587
Bibliography	599
Hints and Answers to Selected Exercises by T. Porco	601
Index	617