Preface

Stability theory is one of the most interesting and important fields of applied mathematics having numerous applications in natural sciences as well as in aerospace, naval, mechanical, civil and electrical engineering. Stability theory was always important for astronomy and celestial mechanics, and during last decades it is applied to stability study of processes in chemistry, biology, economics, and social sciences.

Every physical system contains parameters, and the main goal of the present book is to study how a stable equilibrium state or steady motion becomes unstable or vice versa with a change of problem parameters. Thus, the parameter space is divided into stability and instability domains. It turns out that the boundary between these domains consists of smooth surfaces, but can have different kind of singularities. Qualitatively, typical singularities for systems of ordinary differential equations were classified and listed in [Arnold (1983a); Arnold (1992)]. One of the motivations and challenges of the present book was to bring some qualitative results of bifurcation and catastrophe theory to the space of problem parameters making the theory also *quantitative*, i.e., applicable and practical. It is shown in the book how the stability boundary and its singularities can be described using information on the system.

Behavior of the eigenvalues near the stability boundary with a change of parameters determines stability or instability of the system. Fig. 0.1 reproduced from [Thompson (1982)] shows interaction of eigenvalues for a specific mechanical system, a pipe conveying fluid, depending on a single parameter p. As we can see, the eigenvalues approach each other, collide and diverge with exciting loops and pirouettes making the system stable or unstable. Looking at this and similar figures several questions appear: What are the rules for movements of eigenvalues on the complex plane de-

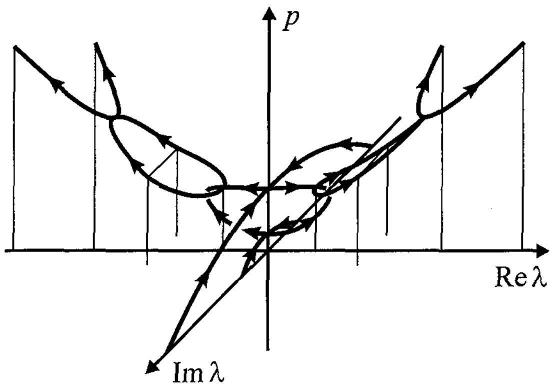

Fig. 0.1 Interaction of eigenvalues for a pipe conveying fluid.

pending on problem parameters? What kind of collisions are possible and which of them are typical? Are there some special properties for behavior of eigenvalues of mechanical systems with symmetry like gyroscopic or conservative systems? What is the relation between eigenvalues and properties of the stability boundary in the parameter space?

In concluding remarks to his book [Bolotin (1963)] pointed out that non-conservative stability problems are closely related to linear non-selfadjoint operators, and it is necessary to develop methods for studying dependence of their eigenvalues on one or more parameters. He also mentioned that general stability properties of linear systems with non-conservative positional (circulatory) forces were not fully investigated, and recalled that in the classical results by [Thomson and Tait (1879)] on stability of mechanical systems circulatory forces were not involved. Bolotin suggested to put more attention to the unexpected effect of destabilization of a circulatory system by small dissipative forces.

It is remarkable that [MacKay (1991)], who derived a formula for movement of simple eigenvalues of a Hamiltonian matrix under non-Hamiltonian perturbation, suggested to generalize the above result for the case of multiple eigenvalues and movement of Floquet multipliers as well as to apply the results to some non-trivial problems.

In this book we present a new multi-parameter bifurcation theory of eigenvalues answering the formulated questions and suggestions. Two important cases of strong and weak interactions (collisions) are distinguished and geometrical interpretation of these interactions is given. First publications on this subject were [Seyranian (1990a); Seyranian (1991a);

Seyranian (1993a)] and here we present an extended and advanced version of the theory. The presence of several parameters and the absence of differentiability of multiple eigenvalues constitute the main mathematical difficulty of the analysis. We could overcome this difficulty studying bifurcations of eigenvalues along smooth curves in the parameter space emitted from the singular points and then analyzing the obtained relations. For the study of bifurcations the perturbation theory of eigenvalues developed in [Vishik and Lyusternik (1960)] turned out to be very useful.

The presented multi-parameter bifurcation theory of eigenvalues is a key point for stability and instability studies. With this theory we analyze singularities of stability boundaries and give a consistent description and explanation for several interesting mechanical effects like gyroscopic stabilization, flutter and divergence instabilities, transference of instability between eigenvalue branches, destabilization and stabilization by small damping, disappearance of flutter instability, parametric resonance in periodically excited systems *etc.*

A significant part of the book is devoted to difficult stability problems of periodic systems dependent on multiple constant parameters. This subject has been a challenge for more than one hundred years since [Mathieu (1868); Floquet (1883); Hill (1886); Rayleigh (1887); Liapunov (1892); Poincaré (1899)]. From the very beginning these problems were multiparameter. For example, finding stable solutions to famous Mathieu–Hill equation is a two-parameter problem. In the present book, with the bifurcation theory of multipliers, geometrical description of the stability boundary and its singularities for periodic systems is given. Then we formulate and solve parametric resonance problems for one- and multiple degrees of freedom systems in three-parameter space of physical parameters: excitation frequency Ω and amplitude δ, and viscous damping coefficient γ under assumption that the two last parameters are small. It is supposed that the unperturbed system is conservative. The main result obtained here is that we find the instability (parametric and combination resonance) domains as half-cones in the three-parameter space with the use of eigenfrequencies and eigenmodes of the corresponding conservative system, see Fig. 0.2. Finally, stability boundaries for non-conservative systems under small periodic excitation are investigated.

As applications of the presented theory, we consider a number of mechanical stability problems including pipes conveying fluid, beams and columns under different loading conditions, rotating shafts and systems

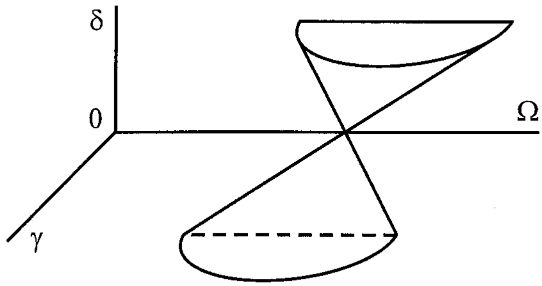

Fig. 0.2 Parametric resonance domain as a half-cone in three-dimensional space of physical parameters.

of connected bodies, panels and wings in airflow *etc.* For these systems we perform the detailed multi-parameter stability analysis showing how the developed bifurcation and singularity theory works in specific problems.

Among previous studies on the stability theory and applications we should mention the books [Liapunov (1892); Chetayev (1961); Bolotin (1963); Bolotin (1964); Panovko and Gubanova (1965); Malkin (1966); Ziegler (1968); Huseyin (1978); Thompson (1982); Huseyin (1986); Leipholz (1987); Yakubovich and Starzhinskii (1987); Troger and Steindl (1991); Kounadis and Kratzig (1995); Merkin (1997); Thomsen (1997); Rumyantsev and Karapetyan (1998)].

The book is based mostly on the authors' personal research, and the relevant papers are given in the list of references. The main results of the book were presented at numerous International Conferences and Symposia. Basic results of the book are given as a one-year course on stability and catastrophes of mechanical systems in Moscow State Lomonosov University by A. P. Seyranian. For the first time this course was presented in Technical University of Denmark and Aalborg University (Denmark) in 1991, see [Seyranian (1991b)]. This course was also given in Bauman Moscow State Technical University in 1993–1994 and Dalian University of Technology (China) in 1994. In 2001 A. P. Seyranian presented six lectures on bifurcations of eigenvalues and stability problems in mechanics at International Centre for Mechanical Sciences in Udine (Italy) [Seyranian and Elishakoff (2002)]. The course on singularities of stability boundaries was given by A. A. Mailybaev in the Institute of Pure and Applied Mathematics IMPA (Brazil) in 2001.

The book is divided into 12 Chapters. Chapter 1 presents an introduction to the stability theory. Chapter 2 is devoted to bifurcation analysis of eigenvalues depending on parameters. This important chapter is used in all parts of the book. In Chapter 3 the stability boundary and its singularities for general systems of ordinary differential equations smoothly dependent on parameters are analyzed. It is shown how to describe singularities in the parameter space using information on the system at the singular point. Chapter 4 presents general bifurcation theory of roots of characteristic polynomials dependent on parameters with application to analysis of stability boundaries. In Chapter 5 we consider linear conservative systems. Change of simple and multiple frequencies depending on several parameters is studied. Multi-parameter stability analysis reveals an interesting relation of singularities of stability boundaries to the so-called bimodal solutions in structural optimization problems. Chapter 6 provides detailed explanation of the effect of gyroscopic stabilization in terms of bifurcation theory of eigenvalues. Chapter 7 studies linear Hamiltonian systems, which are characterized by rich and sophisticated set of different kind of singularities on the stability boundary. Chapter 8 investigates several interesting mechanical phenomena and paradoxes associated with bifurcations and singularities. In Chapter 9 we give an introduction to multi-parameter stability theory of periodic systems. Results of this chapter are used in Chapter 10 for analysis of stability boundaries of general periodic systems. Chapter 11 studies systems with small damping under small periodic excitation, and Chapter 12 considers non-conservative systems under small parametric excitation.

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The book is addressed to graduate students and university professors interested in the stability theory and applications, as well as to researchers and industrial engineers. We hope that the book will promote studies of new problems, effects, and phenomena associated with instabilities and catastrophes, and give a fresh view to classical problems.

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