

Preface

It is a well known fact that university-level mathematics is hard. Just how hard first became apparent to me when I embarked upon my undergraduate studies in mathematics. Nothing had prepared me for the shift in focus from the routine drill-based problem solving of high school mathematics to the intellectual gymnastics that is real mathematics. Luckily I managed to battle my way through the first months of study and gradually began to understand the meaning behind the masses of symbols which constituted the dense sets of formal lectures. As I discovered, mathematics is an amazing and delightful subject awash with possibility, albeit distantly located across a swift and dangerous river of formalism, brevity and logic.

Years later, whilst researching and teaching mathematics, I saw several generations of budding mathematicians battling with the same problems I had initially faced. Naturally, some students were exceptional, and rapidly became skilled mathematicians. Some, unable to make the transition to higher mathematics, quit and pursued mathematics no further. Others, quite successfully, shuffled symbols around and ‘succeeded’ by getting good marks in examinations, without having any meaningful mathematical insight. A fourth category consisted of potentially skilled and bright mathematicians who still found the transition to higher mathematics difficult. Invariably, these were talented students who had come to university without any exposure to higher mathematics whilst at high school. It still surprises me how many students fall into the last two categories.

Upon further investigation, it seemed that there was little in the literature which provided this transitional material in a clear, intuitive and, above all, entertaining manner. On the one hand there are the standard textbooks. Whilst these are, of course, essential, they are also on the whole very dense, difficult and compact affairs. Ill suited for anything except

dedicated study and reference. On the other hand, there are many excellent ‘popular’ books on mathematics. However, these tend to focus on the cutting edge, super-advanced research topics in mathematics, which will be of direct relevance only to a tiny subset of successful mathematicians, and only then after several years of study. Furthermore, these books tend not to include any real mathematical detail; rather like mathematical sight-seeing or intellectual voyeurism, a snapshot of each highlight is to be taken before moving onto the next. Whilst these are great sources of long-term inspiration, or simply an enjoyable read, little mathematical skill is required or imparted by such books.

I felt that surely a compromise between these two extremes could be met: a real mathematics book presented in a more conversational, intuitive and accessible fashion than is usual. For these reasons I was inspired to write this book, a hybrid ‘popular-textbook’; the book I would have loved to own before beginning my career as a mathematician. The goal of the work was simple:

Discover the core elements and highlights of a typical mathematics degree course, in a way which requires only basic high school mathematics to get started. Emphasise the natural beauty and utility of many of these amazing results whilst remaining mathematically honest.

And so the book, after some years of labour, is now finished. I consider it to be suitable for the following groups of people:

- *Aspiring Mathematicians* who want to learn more about the real art of mathematics.
- *Graduates of Mathematics* who would like to read an entertaining ‘highlights’ summary of their university course.
- *Scientists, Engineers* and *Keen Amateurs* who would like to know what it is that mathematicians really get up to.
- *Teachers of Mathematics* who would like a refreshing presentation of the higher material to find examples to inspire themselves and their students.
- Students of *Synoptic* or *Maths for Poets* courses.

As already mentioned: mathematics is hard. This book is no exception; due to the wealth of ideas presented, a high degree of mental effort will be required to read the text. A basic level of knowledge or familiarity with the mathematics detailed in the appendices will be required at various points. However, the book is very conversational and episodic, and may be read

with varying levels of depth. Moreover, the dependence of one topic on the next is kept to a minimum. Wherever possible, each new section starts from the very beginning, so if one area is becoming too difficult or does not interest you, move on to the next. In addition, in order not to disrupt the flow of the text or lose sight of the key underlying mathematical ideas, at some points I have glossed over certain more technical details. Hopefully these points have been clearly indicated, and these omissions should not concern the majority of readers.

Mathematics is an exciting and vibrant art-form. I hope that this book passes on to you some appreciation of the real meaning of mathematics.

Stephen Hewson, April, 2003