

Contents

<i>Foreword</i>	v
<i>Preface</i>	ix
1. Basic Calculus of Variations	1
1.1 Introduction	1
1.2 Euler's Equation for the Simplest Problem	14
1.3 Some Properties of Extremals of the Simplest Functional	19
1.4 Ritz's Method	22
1.5 Natural Boundary Conditions	30
1.6 Some Extensions to More General Functionals	33
1.7 Functionals Depending on Functions in Many Variables .	43
1.8 A Functional with Integrand Depending on Partial Deriva- tives of Higher Order	48
1.9 The First Variation	54
1.10 Isoperimetric Problems	66
1.11 General Form of the First Variation	73
1.12 Movable Ends of Extremals	78
1.13 Weierstrass–Erdmann Conditions and Related Problems .	82
1.14 Sufficient Conditions for Minimum	88
1.15 Exercises	97
2. Elements of Optimal Control Theory	99
2.1 A Variational Problem as a Problem of Optimal Control .	99
2.2 General Problem of Optimal Control	101
2.3 Simplest Problem of Optimal Control	104

2.4	Fundamental Solution of a Linear Ordinary Differential Equation	111
2.5	The Simplest Problem, Continued	112
2.6	Pontryagin's Maximum Principle for the Simplest Problem	113
2.7	Some Mathematical Preliminaries	118
2.8	General Terminal Control Problem	131
2.9	Pontryagin's Maximum Principle for the Terminal Optimal Problem	137
2.10	Generalization of the Terminal Control Problem	140
2.11	Small Variations of Control Function for Terminal Control Problem	145
2.12	A Discrete Version of Small Variations of Control Function for Generalized Terminal Control Problem	147
2.13	Optimal Time Control Problems	151
2.14	Final Remarks on Control Problems	155
2.15	Exercises	157
3.	Functional Analysis	159
3.1	A Normed Space as a Metric Space	160
3.2	Dimension of a Linear Space and Separability	165
3.3	Cauchy Sequences and Banach Spaces	169
3.4	The Completion Theorem	180
3.5	Contraction Mapping Principle	184
3.6	L^p Spaces and the Lebesgue Integral	192
3.7	Sobolev Spaces	199
3.8	Compactness	205
3.9	Inner Product Spaces, Hilbert Spaces	215
3.10	Some Energy Spaces in Mechanics	220
3.11	Operators and Functionals	240
3.12	Some Approximation Theory	245
3.13	Orthogonal Decomposition of a Hilbert Space and the Riesz Representation Theorem	249
3.14	Basis, Gram-Schmidt Procedure, Fourier Series in Hilbert Space	253
3.15	Weak Convergence	259
3.16	Adjoint and Self-adjoint Operators	267
3.17	Compact Operators	273
3.18	Closed Operators	281
3.19	Introduction to Spectral Concepts	285

3.20	The Fredholm Theory in Hilbert Spaces	290
3.21	Exercises	301
4.	Some Applications in Mechanics	307
4.1	Some Problems of Mechanics from the Viewpoint of the Calculus of Variations; the Virtual Work Principle	307
4.2	Equilibrium Problem for a Clamped Membrane and its Generalized Solution	313
4.3	Equilibrium of a Free Membrane	315
4.4	Some Other Problems of Equilibrium of Linear Mechanics	317
4.5	The Ritz and Bubnov–Galerkin Methods	325
4.6	The Hamilton–Ostrogradskij Principle and the General- ized Setup of Dynamical Problems of Classical Mechanics	328
4.7	Generalized Setup of Dynamic Problems for a Membrane	330
4.8	Other Dynamic Problems of Linear Mechanics	345
4.9	The Fourier Method	346
4.10	An Eigenfrequency Boundary Value Problem Arising in Linear Mechanics	348
4.11	The Spectral Theorem	352
4.12	The Fourier Method, Continued	358
4.13	Equilibrium of a von Kármán Plate	363
4.14	A Unilateral Problem	373
4.15	Exercises	380
Appendix A	Hints for Selected Exercises	383
	<i>References</i>	415
	<i>Index</i>	417