

The *Sun Zi Suanjing* (The Mathematical Classic of Sun Zi)

1.1 The text in perspective

The third century AD saw the vigorous growth and brilliant achievements of mathematics in China. The two mathematical texts of antiquity, namely, *Zhou bi suanjing*¹ 周髀算經 (The mathematical classic of the Zhou gnomon) and *Jiu zhang suanshu* 九章算術 (Nine chapters on the mathematical art),² were by then properly annotated by Zhao Junqing 趙君卿 and Liu Hui 劉徽 respectively. The former mathematician highlighted the Pythagorean triplets by supplying geometric-algebraic proofs of their relationships, and the latter elucidated the ancient Chinese mathematical concepts and methods with his own emulations and ramifications (see [Gillon 1977, Swetz & Kao 1977, Ang 1978, Lam & Shen 1984, Wu ed. 1982]). These were followed by *Sun Zi suanjing* 孫子算經 (The mathematical classic of Sun Zi) with the author's preface, which elaborated on the importance and functions of mathematics.

Unlike *Zhou bi suanjing* and *Jiu zhang suanshu* which epitomize the accumulation of mathematical knowledge for a period of time, *Sun Zi*

¹ The romanization of Chinese characters in this book is based on the *hanyu pinyin* 漢語拼音 (Chinese Alphabet Phonetic) system.

² *Jiu zhang suanshu* has been translated into Russian by Berezkina [1957] and into German by Vogel [1968].

suanjing reflects the effort of one who had benefited from the experience of his predecessors. Unlike Liu Hui and others who made their contributions by enriching and embellishing the existing texts, Sun Zi 孫子 saw the need for a basic text aimed at introducing the mathematical tables and the basic approach to mathematical operations. Although *Sun Zi suanjing* could not be compared with *Jiu zhang suanshu* in terms of mathematical content, yet it is able to stand on its own in terms of historical and mathematical significance. It provides a valuable source for understanding the art of computing with rod numerals, and it has the famous remainder problem of indeterminate analysis (Ch. 3, Prob. 26). The text gained official recognition when it was incorporated as one of the ten mathematical classics commissioned for use in the imperial examinations in 656 AD.

The present version of *Sun Zi suanjing* consists of three chapters and a short preface. In the preface, Sun Zi placed mathematics on the pedestal of knowledge and portrayed it as the regulator of nature and mundane affairs. In his eager attempt to highlight the varied functions of mathematics, Sun Zi even said that mathematics could be used “to locate the positions of the celestial and terrestrial spirits” (p. 190). This tinge of the supernatural function of mathematics may be taken as an indication that Sun Zi might indeed have been a monk or a religious scholar.

1.2 Author and date

The identity of Sun Zi³ or Master Sun has yet to be confirmed. In the bibliographical chapters of the *Sui shu* 隋書 (Standard history of the Sui dynasty) (656 AD), the *Jiu Tang shu* 舊唐書 (Old standard history of the Tang dynasty) (945 AD) and the *Xin Tang shu* 新唐書 (New standard history of the Tang dynasty) (1061 AD), *Sun Zi suanjing* was listed, but the author’s name was not mentioned. This shows that as early as the middle of the 7th century, no one seemed to know who Sun Zi was.

It is obvious that Sun Zi had neither high political position nor influential social standing to merit a place in official history. He appeared to be merely a scholar with some Buddhist inclinations. This is evidenced by a problem

³ Zi was an honorific expression given to a respectable person irrespective of whether his full name was known or unknown such as Kong Zi (Confucius), Lao Zi, etc.

on the length of a sutra (Ch. 3, Prob. 4); it is likely that he had in his possession a copy of the sutra as he was familiar with the number of characters in each chapter. Since the introduction of Buddhism to China from the beginning of the Christian era, it was not uncommon for Buddhist monks to be well versed in mathematics. For instance, in the 5th and 6th centuries, a good number of monks and hermits pursued mathematics in seclusion.⁴

Not only was the authorship of *Sun Zi suanjing* difficult to establish, the dating of the text, too, was equally baffling. Several assumptions were based on internal evidence of the text. The mention of Chang'an 長安 in Ch. 3, Prob. 33 confirmed that the text could not have been written during the late Zhou 周 period in the 3rd century BC as assigned by Ruan Yuan 阮元 (1764–1849) or as put forward by Zhu Yizun 朱彝尊 of the 17th century [Qian ed. 1963, p. 275]. This is because Chang'an was first established as a capital and named as such by Emperor Hui 惠, who reigned in the early years of the Han dynasty. The appearance of the problem concerning a Buddhist sutra, which has been mentioned earlier, also confirmed that *Sun Zi suanjing* could not be a pre-Qin text, since Buddhism was not brought to China until the following Han dynasty.⁵

Since there is a problem on the chess board (Ch. 3, Prob. 5) and another on household taxation (Ch. 3, Prob. 9), Yan Dunjie [1937] made a study of the chess game called *wei qi* 圍棋 and the system of taxation. He found that the chess board of 19 lines with a total of 361 positions on the grid and

⁴ An example of such a trend is apparent in the biography of Yin Shao 殷紹 in *Wei shu* 魏書 (Standard history of the Wei dynasty), [Ch. 91, p. 1955].

⁵ Dai Zhen 戴震 (18th century scholar and mathematician) held this view. Ruan Yuan was aware of Dai Zhen's argument in dating the text to the Han dynasty. However he believed that the problem concerning Chang'an and the one on the Buddhist sutra together with others such as the one on the pregnant woman (Ch. 3, Prob. 36) were probably later additions or deliberate amendments of Sun Zi's original version. It was for this reason that he assigned the date of the text to the late Zhou period.

⁶ *Wei qi* literally means "encirclement chess" and another name for it is *go* which is an abbreviation of its Japanese name. The game is still played today; it has become an international game and interest in it is still growing. The game is played on a board which has 19 horizontal lines and 19 vertical lines, forming 361 intersection points. The players use black and white chips or chessmen which are placed at the intersection points. In ancient and medieval China, *wei qi* used to be an intellectual pastime among court officials and scholars.

the system of household taxation had their inception during the middle of the 3rd century AD. Yan inferred from this that *Sun Zi suanjing* could have been a product of the Wei-Jin period (220–420 AD).

In the preface of *Zhang Qiujian suanjing* 張邱建算經 (The mathematical classic of Zhang Qiujian) the author, Zhang Qiujian 張邱建, singled out the method of the problem on “floating of bowls” in *Sun Zi suanjing* (Ch. 3, Prob. 17) for emulation (see [Ang 1969, pp. 120–121]). In the same preface, Zhang also mentioned a problem on “granary” in *Xiahou Yang suanjing* 夏侯陽算經 (The mathematical classic of Xiahou Yang). While the work of Zhang could be ascertained to be written between 468 and 486 AD, that of Xiahou Yang 夏侯陽 remained controversial. In his analysis of the textual content, Qian Baocong [1964, p. 79] commented that apart from the operational rules of arithmetic and the definition on division at the beginning of the text, the rest of *Xiahou Yang suanjing* was probably written by Han Yan 韓延 in the 8th century. The present version does not contain the so-called “granary” problem nor does it carry the commentary of Zhen Luan 甄鸞 (c. 570 AD) as listed in the bibliographical chapters of *Sui shu*, *Jiu Tang shu* and *Xin Tang shu*. When Zhang mentioned the solutions of both Sun Zi’s and Xiahou Yang’s problems, he seemed to refer to them as contemporaries.

Taking into consideration the text of *Sun Zi suanjing* as a whole, Qian [ed. 1963, p. 275] believed that the present version had been tampered with slightly and the original text was probably written at the turn of the 5th century.

In his discussion on the dating of *Sun Zi suanjing*, Wang Ling [1964, p. 489] said that it could not have been written earlier than 280 AD and later than 473 AD. This was because the taxation method by family units (*hu diao* 戶調) in terms of silk floss (*mian* 綿) (Ch. 3, Prob. 9) was established in 280 AD; as for the later date, that was the time when the mensuration scale between *chi* 尺 and *duan* 端 was changed, and Sun Zi followed the old scale (see also [Needham 1959, p. 33]).

1.3 Existing versions

When a department of mathematics was instituted in the National Academy in 656, mathematics was included as a subject in the official examinations. Li Chunfeng 李淳風 was commissioned to edit with annotations a series of

ten mathematical texts for examination purposes. *Sun Zi suanjing* was one of the ten texts known collectively as *Suanjing shi shu* 算經十書 (Ten mathematical classics).

It was noted in the bibliographical chapter of *Xin Tang shu* [Ch. 59, p. 1547] that when Li Chunfeng was writing his commentary, he had in his possession a copy of the text annotated about a century earlier by Zhen Luan 甄鸞. In all subsequent versions of *Sun Zi suanjing*, the front page was almost always preceded by the statement that the text was “officially annotated by Li Chunfeng and others”. However, for reasons unknown to us, Li Chunfeng’s annotations of *Sun Zi suanjing* did not survive except for one comment (see p. 65 & p. 195, fn. 4).

The text with the other official mathematical classics was first published in wood block print by the Imperial Library in 1084 during the Northern Song dynasty. This was the earliest printed version of the text. It was reprinted by Bao Huanzhi 鮑澣之 in 1213 during the Southern Song dynasty.⁷ Later it was also preserved in the *Yongle dadian* 永樂大典 (Great encyclopedia of the Yongle reign), which was compiled between 1403 and 1407.⁸

At the beginning of the Qing dynasty in mid 17th century, a Southern Song printed copy of *Sun Zi suanjing*, dated 1213, was discovered in the family collection of Wang Jie 王杰. This copy was secured by Mao Jin 毛晉 and kept in his Jiguge 汲古閣 library. The copy was subsequently in the possession of Zhang Dunren 張敦仁 (1754–1834). This copy is now preserved in the Library of Shanghai. (See Figs. 1.1 & 1.2.)

It is said that Mao Yi 毛扆, son of Mao Jin, also possessed a handwritten copy of *Sun Zi suanjing* of the Song version. This copy eventually found its way to the Qing palace and is preserved in *Tianlu linlang congshu* 天祿琳琅叢書.

⁷ See Cheng Dawei’s 程大位 *Suanfa tongzong* 算法統宗 (Systematic treatise on arithmetic) (1592) in the section on bibliography.

⁸ There are now only three problems of *Sun Zi suanjing* in the existing fragments of the *Yongle dadian*. These are Ch. 2, Probs. 19 & 20 and Ch. 3, Prob. 14 in *Yongle dadian* [Ch. 16344, pp. 10b & 15a; Ch. 16343, p. 17a].

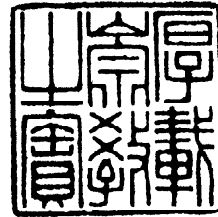


Fig. 1.2 The opening lines of Chapter One of *Sun Zi suanjing*. The first and third seals below the title belongs to Zhang Dunren 張敦仁.

In 1773 when Dai Zhen 戴震 assumed the editing work for the *Siku quanshu* 四庫全書 collection, he extracted and collated the text of *Sun Zi suanjing* from the *Yongle dadian* for inclusion in the encyclopedia compiled between 1773 and 1782 under the patronage of Emperor Qianlong 乾隆. All subsequent versions of *Sun Zi suanjing* such as the *Wu ying dian* 武英殿 block movable-type edition, the *Wei po xie* 微波榭 edition by Kong Jihan 孔繼涵, the *Zhi bu zu zhai* 知不足齋 edition by Bao Tingbo 鮑廷博 and the *Gujin suanxue congshu* 古今算學叢書 edition by Liu Duo 劉鐸 followed Dai Zhen's collated version.

In 1954 Li Yan [1954, pp. 112-125] made some supplementary notes on the text.

1.4 The translation

The translation of *Sun Zi suanjing* in Part Two of this book is based on Qian Baocong's collated version of *Suanjing shi shu* [Qian ed. 1963, pp. 279-322]. Qian [ed. 1963, p. 277] pointed out that when he compared Dai Zhen's collated version with the *Tianlu linlang congshu* edition, he found the former was unsatisfactory as a few of the corrections were unnecessary. The discrepancies among the existing versions were noted by Qian in his annotations on the text; there are 36 notes and we have translated the more significant ones. These are [p. 299, n. 1; p. 303, n. 2, 4, 6 & 7; p. 309, n. 1 & 2; p. 313, n. 1; p. 315, n. 1 & 2; p. 319, n. 2; p. 321, n. 1].