Preface

Many technical systems are described by nonlinear differential equations with discontinuous right-hand sides. Among these are relay automatic control systems, mechanical systems (gyroscopic systems and systems with a Coulomb friction in particular), and a number of systems from electrical and radio engineering.

As a rule, stationary sets of such systems consist of nonunique equilibria. In this book, the methods developed in absolute stability theory are used for their study. Namely, these systems are investigated by means of the Lyapunov functions technique with Lyapunov functions being chosen from a certain given class. The functions are constructed through solving auxiliary algebraical problems, more precisely, through solving some matrix inequalities. Conditions for solvability of these inequalities lead to frequency-domain criteria of one or another type of stability. Frequently, such criteria are unimprovable if the given class of Lyapunov functions is considered.

The book consists of four chapters and an appendix.

In the first chapter some topics from the theory of differential equations with discontinuous right-hand sides are presented. An original notion of a solution of such equations accepted in this book is introduced and justified. Sliding modes are investigated; Lyapunov-type lemmas whose conditions guarantee stability, in some sense, of stationary sets are formulated and proved.

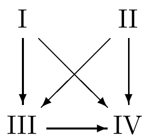
The second chapter concerns algebraic problems arising by the construction of Lyapunov functions. Frequency-domain theorems on solvability of quadratic matrix inequalities are formulated here. The so-called S-procedure, which is a generalization of a method proposed by A.I. Lur'e [Lur'e (1957)], is also justified in this chapter. The origin of these problems is elucidated by the examples of deducing well-known frequency-domain conditions for absolute stability, namely, those of the Popov and circle criteria. The chapter also contains some basic information from the theory of linear control systems, which is used in the book. The proofs of the algebraic statements formulated in Chapter 2 are given in the Appendix.

The third chapter is devoted to the stability study of stationary sets of systems with a nonunique equilibrium and with one or several discontinuous nonlinearities, under various suppositions concerning the spectrum of the linear part. Systems whose discontinuous nonlinearities satisfy quadratic constraints, monotonic, or gradient-type are studied. Frequency-domain criteria for dichotomy (non-oscillation) and for various kinds of stability of equilibria sets are proved.

With the help of the results obtained, dichotomy and stability of a number of specific nonlinear automatic control systems, gyroscopical systems with a Coulomb friction, and nonlinear electrical circuits are investigated.

In the fourth chapter the dynamics of systems with angular coordinates (pendulum-like systems) is examined. Among them are the phase synchronization systems that occur widely in electrical engineering. Such systems are employed in television technology, radiolocation, hydroacoustics, astrionics, and power engineering. The methods of periodical Lyapunov functions, invariant cones, nonlocal reduction, together with frequency-domain methods, are used to obtain sufficient, and sometimes also necessary, conditions for global stability of the stationary sets of multidimensional systems. The results obtained are applied to the approximation of lock-in ranges of phase locked loops and to the investigation of stability of synchronous electric motors.

The dependence diagram of the chapters is the following:



The authors aimed to make the book useful not only for mathematicians engaged in differential equations with discontinuous nonlinearities and the theory of nonlinear automatic control systems, but also for researchers studying dynamics of specific technical systems. That is why much attention has been paid to the detailed analysis of practical problems with the help of the methods developed in the book.

A reader who is interested only in applications may limit himself to reading Sections 2.1 and 1.1, and then pass immediately to Chapter 3.

The basic original results presented in the book are outgrowths of the authors' cooperation; they were reported at the regular seminar of the Division of Mathematical Cybernetics at the Mathematical and Mechanical Department of Saint Petersburg State University.

Chapter 2, Appendix, and Section 1.1 of Chapter 1 were written by V.A. Yakubovich; the rest of Chapter 1 and Chapter 3 were contributed by A.Kh. Gelig; Chapter 4 was written by G.A. Leonov. The final editing was performed by the authors together.

We are greatly indebted to Professor Ardeshir Guran for inviting us to publish this book in his series on Stability, Vibration and Control of Systems. We would like to express our profound gratitude to Professor Michael Cloud for his patient work of bringing the language of the book into accord with international standards and improving a lot of misprints. Our sincere thanks are due to Professor Alexander Churilov for his assistance in typesetting and copyediting and to doctoral student Dmitry Altshuller, whose numerous comments helped us to improve English of the book. We thank the reviewers for their relevant and helpful suggestions.