Chapter 1

Nuclear forces — a review

The motivation and goals for this book have been discussed in detail in the preface. Part 1 of the book is on *Basic Nuclear Structure*, where [Bl52, Bo69, Fe71, Bo75, de74, Pr82, Si87, Ma89, Fe91] provide good background texts.[1] This first chapter is concerned with the essential properties of the nuclear force as described by phenomenological two-nucleon potentials. The discussion summarizes many years of extensive experimental and theoretical effort; it is meant to be a brief *review* and *summary*. It is assumed that the concepts, symbols, and manipulations in this first chapter are familiar to the reader.

1.1 Attractive

That the strong nuclear force is basically attractive is demonstrated in many ways: a bound state of two nucleons, the deuteron, exists in the spin triplet state with $(J^\pi, T) = (1^+, 0)$; interference with the known Coulomb interaction in *pp* scattering demonstrates that the force is also attractive in the spin singlet ${}^1S_0$ state; and, after all, atomic nuclei are self-bound systems.

1.2 Short-range

Nucleon-nucleon scattering is observed to be isotropic, or s-wave with $l = 0$, up to ≈ 10 MeV in the center-of-mass (C-M) system. The reduced mass is $1/\mu_{\text{red}} = 1/m + 1/m = 2/m$. This allows one to make a simple estimate of the range of the

[1]These books, in particular [Pr82], provide an extensive set of references to the original literature. It is impossible to include all the developments in nuclear structure in this part of the book. The references quoted in the text are only those directly relevant to the discussion.

nuclear force through the relations

$$
\begin{aligned}
\hbar l_{\text{max}} &= rp \\
l_{\text{max}} &= r\sqrt{\frac{2\mu_{\text{red}}E}{\hbar^2}} \\
l_{\text{max}} &\approx r(\text{Fermis})\sqrt{\frac{E}{40}\ \text{MeV}}
\end{aligned}
\tag{1.1}
$$

Here we have used the numerical relations (worth remembering)

$$
\begin{aligned}
1\,\text{Fermi} &\equiv 1\,\text{fm} \\
&\equiv 10^{-13}\text{cm} \\
\frac{\hbar^2}{2m_p} &\approx 20.7\ \text{MeV fm}^2
\end{aligned}
\tag{1.2}
$$

A combination of these results indicates that the range of the nuclear force is

$$
r \approx \text{few Fermis} \tag{1.3}
$$

1.3 Spin dependent

The neutron-proton cross section σ_{np} is much too large at low energy to come from any reasonable potential fit to the properties of the deuteron alone

$$
\begin{aligned}
\sigma_{np} &= \frac{3}{4}(^3\sigma) + \frac{1}{4}(^1\sigma) \\
\sigma_{np} &= 20.4 \times 10^{-24}\,\text{cm}^2 \\
&\equiv 20.4\ \text{barns}
\end{aligned}
\tag{1.4}
$$

At low energies, it is a result of effective range theory that the scattering measures only two parameters

$$
k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} r_0 k^2 \tag{1.5}
$$

where a is the scattering length and r_0 is the effective range. The best current values for these quantities for np in the spin singlet and triplet states are [Pr82]

$$
\begin{aligned}
{}^1a &= -23.714 \pm 0.013\ \text{fm} & {}^3a &= 5.425 \pm 0.0014\ \text{fm} \\
{}^1r_0 &= 2.73 \pm 0.03\ \text{fm} & {}^3r_0 &= 1.749 \pm 0.008\ \text{fm}
\end{aligned}
\tag{1.6}
$$

The singlet state just fails to have a bound state ($a = -\infty$), while the triplet state has just one, the deuteron, bound by 2.225 MeV.

1.4 Noncentral

The fact that the deuteron has a nonvanishing quadrupole moment indicates that there must be some $l = 2$ mixed into the $l = 0$ ground state. Therefore the two-nucleon potential cannot be invariant under spatial rotations alone. The most general velocity-independent potential that is invariant under overall rotations and reflections is

$$\begin{aligned} V &= V_0(r) + \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} V_1(r) + S_{12} V_T(r) \\ S_{12} &\equiv \frac{3(\boldsymbol{\sigma_1} \cdot \boldsymbol{r})(\boldsymbol{\sigma_2} \cdot \boldsymbol{r})}{r^2} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \end{aligned} \tag{1.7}$$

The term $S_{12}V_T(r)$ gives rise to the tensor force. Several properties are of interest here:

- Since

$$\begin{aligned} \mathbf{S} &= \frac{1}{2}(\boldsymbol{\sigma_1} + \boldsymbol{\sigma_2}) \\ 4\mathbf{S}^2 &= 4S(S+1) = 6 + 2\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} \end{aligned} \tag{1.8}$$

  It follows that

$$\begin{aligned} \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} &= -3 \qquad ; \text{singlet } (S=0) \\ &= +1 \qquad ; \text{triplet } (S=1) \end{aligned} \tag{1.9}$$

- The total spin S is a good quantum number for the two-nucleon system if the hamiltonian H is symmetric under interchange of particle spins [as in Eq. (1.7)], for then the wave function must be either symmetric ($S = 1$) or antisymmetric ($S = 0$) under this symmetry;[2]
- Higher powers of the spin operators can be reduced to the form in Eq. (1.7) for spin-1/2 particles;
- Since the total spin operator annihilates the singlet state, $(\boldsymbol{\sigma_1} + \boldsymbol{\sigma_2})\,{}^1\chi = 0$, so does the tensor operator S_{12}

$$S_{12}[{}^1\chi] = 0 \tag{1.10}$$

1.5 Charge independent

Charge independence states that the force between any two nucleons is the same $V_{pp} = V_{pn} = V_{nn}$ in the same state. The Pauli principle limits the states that are available to two identical nucleons. For two spin-1/2 nucleons, a complete basis can be characterized by eight quantum numbers, for clearly the states $|\mathbf{p}_1, s_1; \mathbf{p}_2, s_2\rangle$ form such a basis. Alternatively, one can take as the good quantum numbers

[2] If P_σ is the spin exchange operator then $P_\sigma[{}^1\chi(1,2)] \equiv {}^1\chi(2,1) = -{}^1\chi(1,2)$ is odd and, similarly, $P_\sigma[{}^3\chi(1,2)] = +{}^3\chi(1,2)$ is even. Thus from Eqs. (1.9) $P_\sigma = (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/2$.

Table 1.1 States of the two-nucleon system.

States	$^1S_0$	$^1P_1$	$^1D_2$	$^3S_1 + ^3D_1$	$^3P_0$	$^3P_1$	$^3P_2 + ^3F_2$	$^3D_2$
Parity	+	−	+	+	−	−	−	+
Particle exchange	−	+	−	+	−	−	−	+
Particles	*nn* *np* *pp*	*np*	*nn* *np* *pp*	*np*[a]	*nn* *np* *pp*	*nn* *np* *pp*	*nn* *np* *pp*	*np*

[a] The deuteron.

$|E, J, M_J, S, \pi, \mathbf{P}_{\mathrm{CM}}\rangle$. Table 1.1 lists the first few states available to the two-nucleon system. The Pauli principle states that *nn* and *pp* must go into an overall antisymmetric state.[3] Charge independence states that the forces are equal in those states where one can have all three types of particles including *np*; the nuclear force is independent of the charge in these states. At low energy, the cross sections are given in terms of the singlet and triplet amplitudes by

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{np} &= \frac{1}{4}|f(^1S_0)|^2 + \frac{3}{4}|f(^3S_1)|^2 \\ \left(\frac{d\sigma}{d\Omega}\right)_{nn} &= \frac{1}{4} \times 4|f(^1S_0)|^2 = |f(^1S_0)|^2\end{aligned} \tag{1.11}$$

1.6 Exchange character

At higher energies more partial waves contribute to the cross section. At high enough energies, one can use the Born approximation

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{CM}} = \left|\frac{2\mu_{\mathrm{red}}}{4\pi\hbar^2}\int e^{-i\mathbf{q}\cdot\mathbf{r}}V(r)d^3r\right|^2 \tag{1.12}$$

where the momentum transfer $\mathbf{q}$ is defined in Fig. 1.1. For large $\mathbf{q}$ the integrand oscillates rapidly and the integral goes to zero as sketched in Fig. 1.1. The experimental results for *np* scattering are shown in Fig. 1.2. There is significant backscattering, in fact, the cross section is approximately symmetric about 90^o. If $f(\pi - \theta) = f(\theta)$ then only even l partial waves contribute to the cross section; the odd l's will distort $d\sigma/d\Omega$.

To describe this situation one introduces the concept of an exchange force — a force that depends on the symmetry of the wave function.

[3] In terms of isospin we assign $T = 0$ to the states that are even under particle interchange and $T = 1$ to those that are odd, so that the overall wave function is antisymmetric.

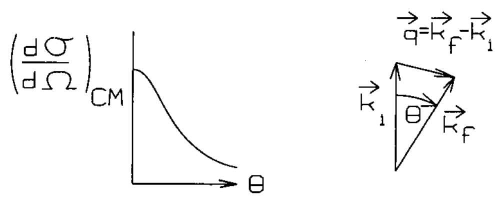

Fig. 1.1. Sketch of cross section in Born approximation.

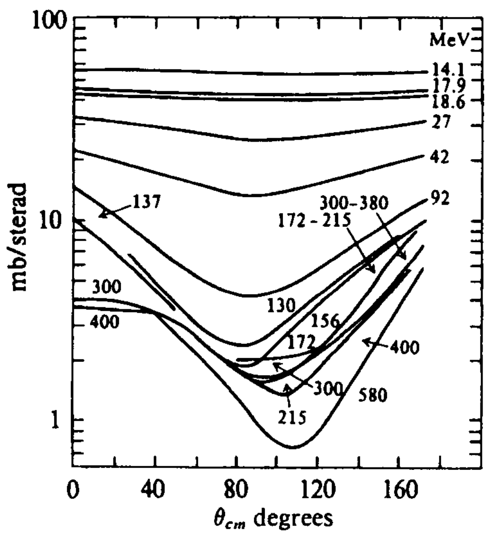

Fig. 1.2. The n–p differential cross section in the C-M system as a function of laboratory energy. From [Pr82].

The interaction is written $V(r)P_{\rm M}$ where the Majorana space exchange operator is defined by[4]

$$P_{\rm M}\phi(\mathbf{r}_2, \mathbf{r}_1) \equiv \phi(\mathbf{r}_1, \mathbf{r}_2) \tag{1.13}$$

Hence since $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$

$$\begin{aligned} P_{\rm M}\phi(\mathbf{r}) &= \phi(-\mathbf{r}) \\ P_{\rm M}Y_{lm}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\right) &= (-1)^l Y_{lm}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\right) \end{aligned} \tag{1.14}$$

The odd l in the amplitude can evidently be eliminated with a Serber force defined

[4]Since the overall wave function is antisymmetric $P_{\rm M}P_\sigma P_\tau = -1$ (Note $P_\sigma^2 = P_\tau^2 = +1$). Thus $P_{\rm M} = -P_\sigma P_\tau = -(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)(1+\vec{\tau}_1\cdot\vec{\tau}_2)/4$ provides an alternate definition.

by

$$V \equiv V(r)\frac{1}{2}(1+P_{\mathrm{M}}) \tag{1.15}$$

The differential cross section in Born approximation with this interaction is

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{CM}} &= \left|\frac{2\mu_{\mathrm{red}}}{4\pi\hbar^2}\int e^{-i\mathbf{k}_f\cdot\mathbf{r}}V(r)\frac{1}{2}(1+P_{\mathrm{M}})e^{i\mathbf{k}_i\cdot\mathbf{r}}d^3r\right|^2 \\ &= \left|\frac{2\mu_{\mathrm{red}}}{4\pi\hbar^2}\int e^{-i\mathbf{k}_f\cdot\mathbf{r}}V(r)\frac{1}{2}(e^{i\mathbf{k}_i\cdot\mathbf{r}}+e^{-i\mathbf{k}_i\cdot\mathbf{r}})d^3r\right|^2 \end{aligned} \tag{1.16}$$

This result is sketched in Fig. 1.3. The nuclear force has roughly a Serber exchange nature; it is very weak in the odd-l states.

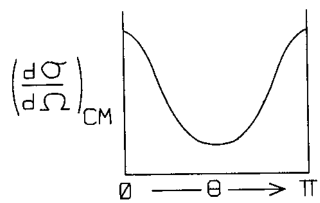

Fig. 1.3. Sketch of cross section in Born approximation with a Serber force.

1.7 Hard core

The *pp* cross section is illustrated in Fig. 1.4.

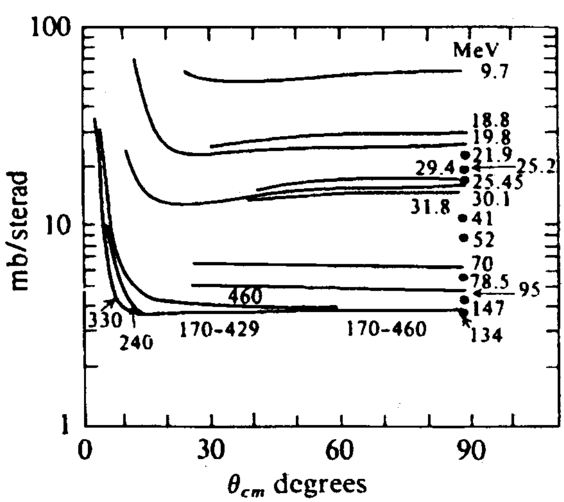

Fig. 1.4. Same as Fig. 1.2 for p–p scattering. From [Pr82].

Recall that since the particles are here identical, one necessarily has the relation $[d\sigma(\pi-\theta)/d\Omega]_{\rm CM} = [d\sigma(\theta)/d\Omega]_{\rm CM}$. Although the cross sections shown in Figs. 1.2 and 1.4 are very different, it is possible to make a charge-independent analysis of *np* and *pp* scattering as first shown in detail by Breit and coworkers [Br39, Se68]. The overall magnitude of the *pp* cross section indicates that more than s-wave nuclear scattering must be included (recall the unitarity bound of π/k^2), and the higher partial waves must interfere so as to give the observed flat angular distribution beyond the Coulomb peak. A hard core will change the sign of the s-wave phase shifts at high energy and allow the $^1S - {}^1D$ interference term in *pp* scattering to yield a uniform angular distribution as first demonstrated by Jastrow [Ja51]; with a Serber force, it is only the states $(^1S_0, {}^1D_2)$ in Table 1.1 that contribute to nuclear *pp* scattering. Recall that for a pure hard core potential the s-wave phase shift is negative $\delta_0 = -ka$ as illustrated in Fig. 1.4.

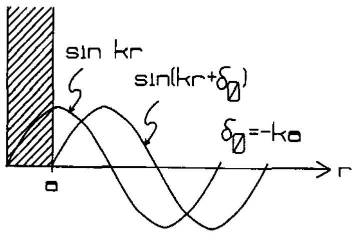

Fig. 1.5. The s-wave phase shift for scattering from a hard-core potential.

With a finite attractive well outside of the hard core, one again expects to see the negative phase shift arising from the hard core at high enough energy. The experimental situation for the s-wave phase shifts in both *pp* and *np* scattering is sketched in Fig. 1.6.

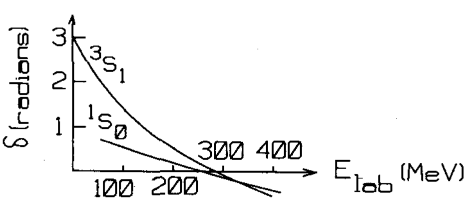

Fig. 1.6. Sketch of s-wave nucleon-nucleon phase shifts. After [Pr82].

From an analysis of the data, one concludes that there is a hard core[5] of radius

$$r_c \approx 0.4 \text{ to } 0.5 \text{ fm} \tag{1.17}$$

in the relative coordinate in the nucleon-nucleon interaction.

[5]Or, more generally, a strong, short-range repulsion.

1.8 Spin-orbit force

It is difficult to explain the large nucleon polarizations observed perpendicular to the plane of scattering with just the central and tensor forces discussed above. To explain the data one must also include a spin-orbit potential of the form

$$\begin{aligned} V &= -V_{\mathrm{SO}}\mathbf{L}\cdot\mathbf{S} \\ \mathbf{L}\cdot\mathbf{S} &= \frac{1}{2}[J(J+1) - l(l+1) - S(S+1)] \end{aligned} \tag{1.18}$$

This last expression vanishes if either $S = 0$ ($l = J$) or $l = 0$ ($S = J$). The spin-orbit force vanishes in s-states and is empirically observed to have a short range; thus it is only effective at higher energies.

1.9 Summary

The present situation with respect to our phenomenological knowledge of the nucleon-nucleon force is the following:

- The experimental scattering data can be fit up to laboratory energies of $\approx$ 300 MeV with a set of potentials depending on spins and parities ${}^1V_C^+$, ${}^3V_C^+$, ${}^1V_C^-$, ${}^3V_C^-$, ${}^3V_T^+$, ${}^3V_T^-$, etc;
- The potentials contain a hard core with $r_c \approx 0.4$ to 0.5 fm;[6]
- The forces in the odd-l states are relatively weak at low energies, and on the average slightly repulsive;
- The tensor force is necessary to understand the quadrupole moment of the deuteron (and its binding);
- A strong, short-range, spin-orbit force is necessary to explain the polarization at high energy.

Commonly used nucleon-nucleon potentials include the "Bonn potential" in [Ma89], the "Paris potential" [La80], and the "Reid potential" [Re68]. The first two contain the one-meson (boson) exchange potentials (OBEP) at large distances.

1.10 Meson theory of nuclear forces

The exchange of a neutral scalar meson of Compton wavelength $1/m \equiv \hbar/mc$ (Fig. 1.7) in the limit of infinitely heavy sources gives rise to the celebrated

[6] Although a hard core provides the way to represent this short-range repulsion within the framework of static two-body potentials, a short-range velocity-dependent potential that becomes repulsive at higher momenta leads to similar results [Du56]. We shall see in chapter 14 that the latter description is obtained as an immediate consequence of relativistic mean field theory.

Yukawa potential [Yu35]

$$V(r) = -\frac{g^2}{4\pi c^2}\frac{e^{-mr}}{r} \tag{1.19}$$

A derivation of this result, as well as the potentials arising from other types of meson exchange, is given in appendix A.1.

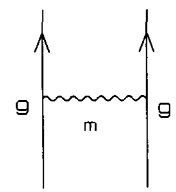

Fig. 1.7. Contribution of neutral scalar meson exchange to the N–N interaction.

In charge-independent pseudoscalar meson theory with a nonrelativistic coupling of $\boldsymbol{\tau}(\boldsymbol{\sigma}\cdot\nabla)$ at each vertex, one obtains a tensor force of the correct sign in the N–N interaction. In fact, for this reason, Pauli [Pa48] claimed there had to be a long-range pseudoscalar meson exchange before the π-meson was discovered. Since the π is the lightest known meson, the 1–π exchange potential is exact at large distances $r \to \infty$; mesons with higher mass $\bar{m}$ give a potential that goes as $e^{-\bar{m}r}/r$ by the uncertainty principle. The existence of this 1–π exchange tail in the N–N interaction has by now been verified experimentally in many ways.

The Paris and Bonn potentials [La80, Ma89] include the exchange of $(\pi, \sigma, \boldsymbol{\rho}, \omega)$ mesons with spin and isospin $(J^\pi, T) = (0^-, 1), (0^+, 0), (1^-, 1), (1^-, 0)$, respectively, in the long-range part of the N–N potential. The short-distance behavior of the interaction is then parameterized.

One can get a qualitative understanding of the short-range repulsion and spin-orbit force in the strong N–N interaction by considering meson exchange and using the analogy with quantum electrodynamics (QED). Suppose one couples a neutral vector meson field, the ω, to the conserved baryon current. Then just as with the Coulomb interaction in atomic physics, which is described by the coupling of a neutral vector meson field (the photon) to the conserved electromagnetic current:

- Like baryonic charges repel;
- Unlike baryonic charges (e.g., p–$\bar{p}$) attract;
- There will be a spin-orbit force;
- While the range of the Coulomb potential $1/r$ is infinite because the mass of the photon vanishes $m_\gamma = 0$, the range of the strong nuclear effects will be $\sim \hbar/m_\omega c$. Since the ω has a large mass, the force will be short-range.

The meson exchange theory of the nuclear force and its consequences are well summarized in [Ma89].

To get ahead of ourselves, there is now a theory of the strong interactions based on an underlying structure of *quarks.* The observed strongly interacting *hadrons*, mesons and nucleons, are themselves composites of quarks. The quarks interact through the exchange of gluons in a theory known as quantum chromodynamics (QCD). The quarks, gluons, and their interactions are confined to the interior of the hadrons. It is still true that the long-range part of the nuclear force must be described by meson exchange. How can one understand this? The key was provided by Weinberg [We90, We91]. In the low-energy nuclear domain, one can write an effective field theory in terms of hadrons as the generalized coordinates of choice. This theory must reflect the underlying symmetry structure of QCD [Be95]. Expansion in appropriate small dimensionless parameters (for example, q/M in the nucleon-nucleon case), and a fit of coupling constants to experiment, then allow one to systematically compute other observables. This approach puts the meson theory of the nuclear force on a firm theoretical foundation, at least in the appropriate range of the expansion parameters. Application of this effective field theory approach to the N-N force can be found in [Or92, Or94, Or96]. The very large scattering lengths in N-N scattering put another characteristic length in the problem and one must be careful in making the proper expansions [Ka96, Ka98, Ka98a]. Results from this effective field theory approach are quite satisfying [Ep00]. Of course, given an effective lagrangian one can proceed to calculate other quantities such as the three-nucleon force, that is, the force present in addition to the additive two-body interactions when three nucleons come together [We92a, Fr99, Ep02]. The up-to-date developments in the theory of two- and three-nucleon forces can always be found in the proceedings of the most recent International Conference on Few-Body Physics (e.g. [Fe01]).[7]

We shall spend a large part of the remainder of this book on effective hadronic field theory and QCD. For now, we return to some basic elements of nuclear structure which ultimately reflect their consequences.

[7] Recent nucleon-nucleon potentials can be found in [Ma87, Ma89, St94, Wi95, Or96, Ma01, En03].