Chapter 1

Introduction

The interaction between a bridge and the vehicles moving over the bridge is a coupled, nonlinear dynamic problem. Conventionally, most research has been focused on the dynamic or *impact* response of the bridge, but not of the moving vehicles. For the cases where only the bridge response is desired, the moving vehicles have frequently been approximated to the extreme as a number of moving loads. However, whenever the responses of both the bridge and moving vehicles are desired, as encountered in the design of high-speed railways, models that can adequately account for the dynamic properties of the moving vehicles should be adopted. In this chapter, the key factors involved in the dynamic interaction between the bridge and moving vehicles will be discussed, along with procedures for solving the vehicle–bridge interaction problems. The materials presented in this chapter have been revised from the review paper by Yang and Yau (1998) with supplement of the relevant literature published recently.

1.1. Major Considerations

The dynamic interaction between a bridge and the moving vehicles represents a special discipline within the broad area of structural dynamics. The vehicles considered may be those constituting the traffic flow of a highway bridge, in general, or those that form a connected line of railroad cars, in particular. From the theoretical point of view, the two subsystems, i.e., the bridge and moving vehicles, can be simulated as two elastic structures, of which each is characterized by some frequencies of vibration. The two subsystems interact with each other through the *contact forces*, i.e., the forces induced at the contact points between the wheels and rails surface (of the railway bridge) or pavement surface (of the highway bridge). A problem such as this is nonlinear and time-dependent due to the fact that the contact forces may move from time to time, while their magnitudes do not remain constant, as a result of the relative movement of the two subsystems. The way by which the two subsystems interact with each other is determined primarily by the inherent frequencies of the two subsystems and the driving frequency of the moving vehicles. In this book, we prefer to use the term *vehicle–bridge interaction* (VBI) to refer to the interaction between the two subsystems. The vehicle considered in this book is a general term, which can be a car, a truck, a tractor-trailer, or a railroad car that forms part of the train. The term bridge is also a general one. It can be a simply-supported beam, a multi-span continuous beam, or a bridge of any types used in highways and railways, with or with no account of the effects of surface pavement (for highways) or rails and ballast (for railways). The consideration of the VBI is necessary if the vehicle response, in addition to the bridge response, is desired. In the design of high-speed railway bridges, for instance, the maximum vertical and/or lateral accelerations of the moving vehicles are used as indicators for evaluating the riding comfort of passengers carried by the train. Besides, the vertical and lateral contact forces of the wheels of railroad cars with the rails represent a kind of information central to assessment of the risk of derailment for moving trains, especially in the presence of earthquake shaking.

In many cases, especially when the vehicle to bridge mass ratio is small, the elastic and inertial effects of the vehicles may be ignored and much simpler models can be adopted for the vehicles. One typical example is the simulation of a moving vehicle over a bridge as a single moving load, which has been conventionally referred to as the *moving load* model (Fig. 1.1). Since the interaction between the two subsystems has been ignored, the moving load model is good only for computing the response of the larger subsystem, i.e., the bridge,

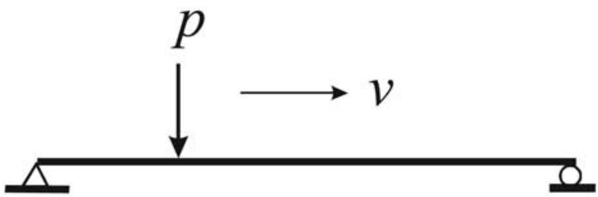


Fig. 1.1. Moving load model.

but not of the smaller subsystem, i.e., the vehicle. In this book, the moving load problem can be regarded as a special case of the more general formulation that considers the various dynamic properties of the moving vehicles.

The objective of this book is to establish some efficient methods within the framework of finite element methods for solving the dynamic response of the VBI systems. The formulation of these methods will be kept as general as possible, so that they can be applied to most conceivable problems. However, in deriving the fundamental theories using the analytical approaches or in conducting the parametric studies to illustrate the various dynamic effects involved, more emphasis will be placed on the problems encountered in the design of high-speed railway bridges, so as to reflect the public concern over the safety and the riding comfort of high-speed trains. It is believed that the methodologies established herein can be applied to solving similar problems encountered in traditional railways and mass rapid transit systems.

From the point of view of structural dynamics, a railway bridge is different from a highway bridge in that the sources of excitation caused by the moving vehicles are different for the two cases. For example, the vehicles moving over a highway bridge are random in nature. The vehicles constituting the highway traffic may vary in terms of the axle weight, axle interval, moving speed, and even the headway. However, a train moving over a railway bridge can generally be regarded as a sequence of identical vehicles in connection, plus one or two locomotives. Conventionally, a train has been simplified as a sequence of moving masses, or in the extreme case as a sequence of concentrated loads, of regular intervals. Because of the repetitive character of the wheel or bogie loads, a moving train usually contains some inherent frequencies, plus an excitation frequency associated with the moving speed. If any of these frequencies coincides with any of the frequencies of vibration of the bridge, the so-called *resonance* phenomenon will be induced on the bridge by the moving train, in the sense that the response will be continuously built up, as there are more railroad cars passing the bridge. Under the condition of resonance, great amplification in the bridge responses, as well as in the vehicles response, can be expected, which is likely to affect the life span of the bridge and the riding quality of running vehicles. It is advisable that the phenomenon of resonance be circumvented from the onset in the design of railway bridges.

Research on the dynamic response of bridges caused by the vehicular movement dates back to the mid-nineteenth century, following primarily the works of Willis (1849) and Stokes (1849) in investigating the collapse of the Chaster Rail Bridge in England in 1847, the first case for the collapse of a railway bridge in history. In these pioneer works, the effect of inertia of the beam was ignored, and the vehicle is modeled as a concentrated moving mass traveling at constant speeds. Although for this particular case, an exact solution can be obtained, its applicability remains rather limited due to the omission of the inertial effect of the beam. Nevertheless, the contribution of Stokes and Willis is considered historical, since they are among the first to bring the problem of vehicle impacts to the design desks of bridge engineers.

In the past two decades, the amount of research conducted on the vibration of bridges under moving vehicles has been increasing at a rate much faster than ever, partly due to the successful operation of high-speed railways in Japan and some European countries. It is difficult, if not impossible, to have a complete count of all the works conducted by previous researchers on this subject. For the days when hand calculations and slide rules play the most important role in design offices, i.e., before the advent of digital computers in the 1940s, investigations on bridge dynamics were concerned mainly with the development of analytical or approximate solutions for some simple, fundamental problems. Researchers of this period who were frequently cited in the literature include Timoshenko (1922), Jeffcott (1929) and Lowan (1935). The work by Inglis (1934) contains an early general treatment on the dynamics of railway bridges, which also lays the foundation for the following development.

The advent of digital computers, later followed by workstations, has enabled researchers to adopt more realistic bridge and vehicle models in analysis. The general texts by Timoshenko and Young (1955) and Biggs (1964) on structural dynamics contain some partial treatment on the moving load problems. Other texts that should be mentioned include the one by Frýba (1972) in analyzing the vibration of structures under moving loads, and those by Garg and Dukkipati (1984) and Frýba (1996) in dealing with the vibration of railway bridges. Starting from 1975, literature reviews were conducted by Ting and co-workers from time to time to update the related researches on vehicle–guideway interactions (Ting *et al.*, 1975; Genin and Ting, 1979; Ting and Genin, 1980; Ting and Yener, 1983; Taheri *et al.*, 1990). Nowadays, very powerful numerical methods, especially those based on the finite element methods, can be employed to analyze the dynamic behavior of bridges and moving vehicles, with virtually no limit placed on the level of complexity of the models used for the two subsystems. It should be noted that most of the works mentioned above were concerned primarily with the vibration of the bridge or supporting structure, but not of the moving vehicles.

1.2. Vehicle Models

By neglecting the inertia effect of the vehicle and considering a vehicle as a moving load or pulsating force, Timoshenko (1922) derived an enormous number of approximate solutions to the problem of simple beams under moving loads. Similar models were adopted by Ayre *et al.* (1950) and Ayre and Jacobsen (1950) in studying the dynamic responses of a two-span beam, and later by Vellozzi (1967) in studying the vibration of suspension bridges. The moving load model was also adopted by Chen (1978) in analyzing the dynamic response of continuous beams. Research on the vibration of bridges traveled by moving loads is abundant. It is only possible to cite a few of the most related ones, for instance, the works by Tan and Shore (1968a), Frýba (1972), Fertis (1973), Sridharan and Mallik (1979), Wu and Dai (1987), Weaver *et al.* (1990), Galdos *et al.* (1993), Gbadeyan and Oni (1995), Wang (1997), Zheng *et al.* (1998), Rao (2000), Chen and Li (2000), and Dugush and Eisenberger (2002), among others.

The *moving load* model is the simplest model that can be conceived, which has been frequently adopted by researchers in studying the vehicle-induced bridge vibrations. With this model, the essential dynamic characteristics of the bridge caused by the moving action of the vehicle can be captured with a sufficient degree of accuracy. However, the effect of interaction between the bridge and the moving vehicle was just ignored. For this reason, the moving load model is good only for the case where the mass of the vehicle is small relative to that of the bridge, and only when the vehicle response is not of interest.

For cases where the inertia of the vehicle cannot be regarded as small, a *moving mass* model (Fig. 1.2) should be adopted instead. The inertial effects of both the beam and the moving vehicle were studied as early as in 1929 by Jeffcott (1929) by the method of successive approximations. The investigations along this line were later carried out by a number of researchers. Stanišić and Hardin (1969) determined the response of a simple beam under an arbitrary number of moving masses by employing the Fourier series expansion. By the use of Green's function, algorithms for dealing with the moving mass problem has been studied by Ting *et al.* (1974) and Sadiku and Leipholz (1987). For a simple beam carrying a single moving mass, an exact, closed form solution was derived by Stanišić (1985) by means of expansion of the eigenfunctions in a series. The same

Fig. 1.2. Moving mass model.

moving mass model was adopted by Akin and Mofid (1989) in their study of the dynamic response of beams with various boundary conditions using an analytical–numerical approach.

One drawback with the moving mass model is that it excludes consideration of the bouncing action of the moving mass relative to the bridge. Such an effect is expected to be significant in the presence of rail irregularities or pavement roughness, or for vehicles moving at rather high speeds. Occasionally, it may be necessary to consider the separation and recontact of the moving vehicle with the bridge for some very bad road conditions, in which the bouncing action of the vehicles plays a decisive role in the seperation–recontact process.

The vehicle model can still be enhanced through consideration of the elastic and damping effects of the suspension systems. The simplest model in this case is a moving mass supported by a spring-dashpot unit, the so-called *sprung mass* model (Fig. 1.3). Biggs (1964) presented a semi-analytical solution to the problem of a simple beam traversed by a sprung mass. By using the series expansion technique, Pesterev *et al.* (2001) examined the response of an elastic continuum to multiple moving oscillations. Later, Pesterev *et al.* (2003) studied in depth the asymptotics of the solution of the moving oscillator problem and found that in the limiting case the moving oscillator problem and the moving mass problem for a simply supported beam are equivalent in terms of the beam displacements, but not in terms of the beam stresses. Also, it was shown that for small values of spring stiffness, the moving oscillator problem is equivalent to the moving load problem. In the book by Frýba (1972), a comprehensive treatment was given for the various vehicle models,

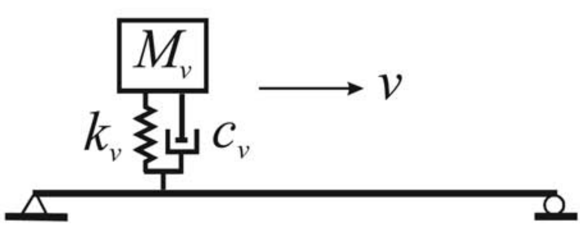


Fig. 1.3. Sprung mass model.

i.e., the moving load, moving mass, and moving sprung mass, concerning primarily the dynamic response of the structure traveled by vehicles. The analytical solutions as well as numerical solutions for some problems were presented in this book.

Because of the emergence of high-performance computers and the advance in computation technologies, it becomes feasible to have a more realistic modeling of the dynamic properties of the various components constituting a moving vehicle. Previously, the elastic effect of the tires and suspension mechanisms has been modeled by springs, the damping effect of the tires, suspension systems, and air-cushion by dashpots (Tan and Shore, 1968b; Genin *et al.*, 1975; Blejwas *et al.*, 1979; Genin and Chung, 1979; Humar and Kashif, 1993; Green and Cebon, 1994), and the energy dissipating effect of the interleaf mechanism by frictional devices (Veletsos and Huang, 1970; Chatterjee *et al.*, 1994; Tan *et al.*, 1998). Using such techniques, a multiple-axle truck or tractor-trailer can be represented as a number of discrete masses each supported by a set of spring and dashpot or frictional device. In the study by Yang *et al.* (1999), a railroad car was simulated as a rigid beam supported by two sets of spring-dashpot unit each resting on a wheel mass. Such a model enables us to consider the pitching effect of the car body.

To represent the various dynamic properties of railway freight cars, vehicle models that contain dozens of degrees of freedom (DOFs) have been devised and used by Chu *et al.* (1986), Wang *et al.* (1991), Xia *et al.* (2000), and Zhang *et al.* (2001a). In order to study the train–rails–bridge interaction, a train composed of a sequence of identical cars was considered by Wu *et al.* (2001), in which each car is assumed to consist of a car body, assumed to be rigid, resting on the front and rear bogies, each of which in turn is supported by two wheelsets. A total of 5 DOFs was assigned to the car body and also to each bogie, to account for the vertical, lateral, rolling, yawing, and pitching motions. In contrast, only three DOFs are assigned to each wheelset, which relate to the vertical, lateral and rolling motions.

Although the use of a more sophisticated vehicle model can make the simulation more realistic, it does create certain computation problems. For instance, in the simulation of bridges subjected to a series of railroad cars or highway vehicles that appear as a random flow (Yang *et al.*, 1996), divergence or slow convergence may occur in the process of iteration searching for a large number of contact forces at the wheels/rails or wheels/girder contact points in a step-by-step time-history analysis. The other concern here is that using simplified models can help identify the key parameters dominating the dynamic response of the bridge, which is beneficial for the developing of rational formulas for use in the design codes (Humar and Kashif, 1993).

1.3. Bridge Models

A beam that is simply-supported at both ends is the most popular structure that has ever been adopted in the study of vehicle-induced vibrations. Except for the research works that rely exclusively on analytical approaches, there is basically no restriction on the type of structures considered for the VBI problems, as the structures can always be represented by finite elements of various forms; the only difference being that a simpler bridge model requires less preparation and computation efforts.

In the past, various types of bridges have been considered in study of the vehicle-induced vibrations, which include the truss bridges (Chu *et al.*, 1979; Wiriyachai *et al.*, 1982), multispan uniform or nonuniform bridges (Wu and Dai, 1987; Yang *et al.*, 1995; Kou and DeWolf, 1997; Cheung *et al.*, 1999; Marchesiello *et al.*, 1999), girder or multigirder bridges (Chu *et al.*, 1986; Hwang and Nowak, 1991; Huang *et al.*, 1993; Cai *et al.*, 1994), continuous beams (Wu and Dai, 1987; Yang *et al.*, 1995), curved girder bridges (Tan and Shore, 1968a,b; Galdos *et al.*, 1993; Chang, 1997; Yang *et al.*, 2001), guideways (Genin *et al.*, 1975), steel plate girder bridges (Kawatani and Kim, 2001), and arch bridges (Chatterjee and Datta, 1995; Ju and Lin, 2003). The impact factor of horizontally-curved box bridges was studied by Galdos *et al.* (1993) and Senthilvasan *et al.* (2002). The dynamic response of a flat plate under the moving load was studied by Wu *et al.* (1987).

The dynamic response of cable-stayed bridges to moving vehicles has been studied by a number of researchers. By simulating the cable-stayed bridge as a beam resting on an elastic foundation, Meisenholder and Weidlinger (1974) proposed an approach for modeling the dynamic effects of cable-stayed guideways subjected to track levitated vehicles moving at high speeds. The effect of road surface roughness was considered by Wang and Huang (1992) in studying the cable-stayed bridge vibrations. By using an approximate bridge model, taking into account the nonlinear effect of cables, the dynamic response of cable-stayed bridges under moving loads was analyzed by Yang and Fonder (1998). In the review paper by Diana *et al.* (2000) for the railway runability of long-span cable supported bridges, it was noted that the impact effect of cable-stayed bridges is more sensitive than that of suspension bridges. Recently, Au *et al.* (2001a,b) investigated the impact effects of cable-stayed bridges under railway traffic using various vehicle models, and concluded that the moving force and moving mass models significantly underestimate the impact effects and the effects of random road surface roughness on the impact response of the bridge deck are more significant at sections close to the bridge towers. Guo and Xu (2001) studied the interaction between a cable-stayed bridge and a tractor-trailer moving over the bridge by a fully computerized approach. Recently, a hybrid tuned mass damper system composed of several subsystems was proposed by Yau and Yang (2004) for suppressing the multiple resonant peaks of cable-stayed bridges that may be excited by high-speed trains.

The vibration of suspension bridges under the vehicular movement was investigated by Chatterjee *et al.* (1994) with the torsional vibration taken into account. The dynamic interaction between a long suspension bridge, which has a main span length of 1377 m, and the running train was shown to be insignificant by Xia *et al.* (2000). The same suspension bridge was later studied by Xu *et al.* (2003), considering that there are high winds acting on the bridge, but not directly on the running train; the latter being protected from exposure to the high wind. Their results indicated that the wind-induced vibration on the bridge is detrimental to the running safety of the train and also to the riding comfort of passengers.

Another concern in simulation of the bridge response has been the inclusion of road *surface roughness* or *rail irregularities*. It has been reported that road surface or pavement roughness can significantly affect the impact response of bridges (Paultre *et al.*, 1992). However, the elevation of roughness or surface profile depends primarily on the workmanship involved in the construction of pavement or rail tracks and on how they are maintained, which, though random in nature, may contain some inherent frequencies. In most cases, the surface roughness or rail irregularities, which is three-dimensional in nature, is often approximated by a two-dimensional profile. As for the railways, it is realized that the profiles of irregularities on the two rails of a track may be different.

The road surface roughness was considered by Gupta (1980) by representing the elevation of road surface by a sine function. To account for its random nature, the road profile can be modeled as a stationary Gaussian random process and generated using certain power spectral density functions. Methods similar to this have been widely adopted by researchers in studying the vehicle-induced bridge vibrations (Inbanathan and Wieland, 1987; Coussy *et al.*, 1989, Hwang and Nowak, 1991; Chatterjee *et al.*, 1994; Chang and Lee, 1994; Henchi *et al.*, 1998; Pan and Li, 2002). The power spectral density functions developed by Dodds and Robson (1973) have been modified and used by Wang and Huang (1992) and Huang *et al.* (1993) in their analyses. The work by Marcondes *et al.* (1991) is of interest in that the power spectral density functions used to compute the road elevation have been determined by using the data collected from a field measurement, with distinction made for three different categories of pavement. Such an approach was adopted by Yang and Lin (1995) in the study of simple and continuous beams traveled by vehicles moving at different speeds.

As far as railway bridges are concerned, track irregularities may occur as a result of initial installation errors, degradation of support materials, and dislocation of track joints. Four geometric parameters can be used to quantitatively describe the rail irregularities, i.e., the vertical profile, cross level, alignment, and gauge (Wiriyachai *et al.*, 1982; Chu *et al.*, 1986; Wang *et al.*, 1991). From the point of structural dynamics, it is the wavelengths or frequencies implied by the rail irregularities that are crucial to the dynamic behavior of the VBI system. The frequencies implied by the surface roughness of a bridge plays a role similar to that of the bridge frequencies, in that resonance may occur on the bridge and traversing vehicles, if any of the excitation or vehicle frequencies coincides with, or are close to, any of the frequencies implied by the surface roughness.

1.4. Railway Bridges and Vehicles

Most of the research works cited above consider only a single or very small number of vehicular loads. In contrast, comparatively few works have been conducted on the dynamic response of bridge structures under the action of a sequence of moving loads with regular intervals, to simulate the effect of a connected line of train loads (Fig. 1.4). Bolotin (1964) studied a beam subjected to an infinite sequence of equal loads with uniform interval d and constant speed v. In his study, the period d/v of the moving loads has been identified as a key parameter. For the same problem, Frýba (1972) concluded that the response of the forced steady-state vibration will attain its maximum when the time intervals between two successive moving loads are equal to some periods of vibration of the beam in free vibration or to an integer multiple thereof. Kurihara and Shimogo (1978a,b) investigated the vibration and stability problems of a simple beam subjected to a series of discrete moving loads. The dynamic response of a girder or truss bridge during the passage of a series of railway vehicles was studied by Chu *et al.* (1979). By the transfer matrix method, Wu and Dai (1987) studied the response of multispan nonuniform beams subjected to two sets of identical loads moving in the same or opposite directions. Savin (2001) derived an analytical expression of the dynamic amplification factor and response spectrum for beams with various boundary conditions under successive moving loads.

Partly enhanced by the successful operation of high-speed railways worldwide, the dynamic response of railway bridges is receiving much more attention from researchers than ever. Matsuura (1976)

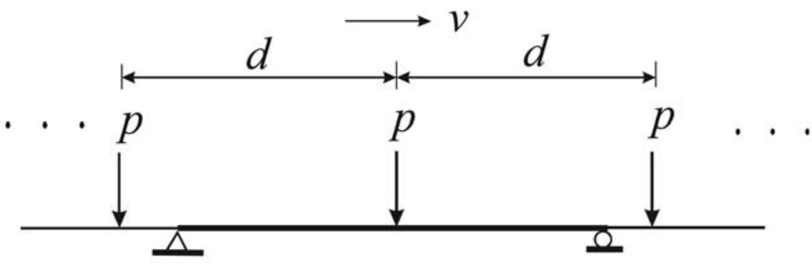


Fig. 1.4. Railway vehicles in series.

studied the dynamic behavior of various bridge girders used in the Shinkansen system. With the rail irregularities represented by power spectral densities, the impact responses of various railway bridges were investigated by Wiriyachai *et al.* (1982) and Chu *et al.* (1986). Following a brief review of the state-of-the-art methodologies for simulating the train–bridge interactions, Diana and Cheli (1989) studied the dynamic behavior of a train running over a long span bridge. By modeling a vehicle as a moving force or sprung mass, Cai *et al.* (1994) investigated the dynamic characteristics of single- and two-span beams subjected to vehicles moving at high speeds.

With the advancement in locomotive and control technologies, railway trains that have a design speed of 350 km/h or higher are not uncommon nowadays. In the literature, a maximum speed of 515.3 km/h has been reported during a field test (Delfosse, 1991). As far as high-speed trains are concerned, one needs to consider not only the vibration amplitudes of the bridge, but also the riding comfort of passengers carried by the trains, which can be assessed from the vertical or lateral accelerations of the moving vehicles (Diana and Cheli, 1989; Yau *et al.*, 1999). Due to the relatively stringent requirements imposed on the allowable deflection of the bridge and on the riding comfort of moving vehicles, the design of high-speed railway bridges is generally governed by the conditions of serviceability, rather than by strength and yielding, as learned from the design practices in Taiwan.

Recently, the dynamic response of a typical bridge under the passage of various commercial high-speed trains was studied by Hsu

(1996), Yau (1996), Chang (1997), and Wu (2000). In particular, the effect of column stiffness on the dynamic response of bridges traveled by high-speed trains was studied by Hsu (1996), and the effects of ballast and elastic bearings by Yau (1996). It was demonstrated by Yau *et al.* (2001) that the insertion of elastic bearings at the supports of bridge girders for the purpose of isolating the earthquake forces may adversely amplify the dynamic response of the beam to moving train loads. Museros *et al.* (2002) investigated the influence of sleepers and ballast layers, as well as train–bridge interactions, on the response of short high-speed railway bridges. They concluded that inclusion of these factors can result in smaller maximum displacements and accelerations on the bridge, compared with those obtained using barely the moving loads model. The mechanism involved in the phenomena of resonance and cancellation for elastically supported beams was further explored in the study by Yang *et al.* (2004).

Other related effects that have been investigated include the torsional vibration of bridges caused by vehicles moving along one of the two tracks on a bridge, the crossing of two vehicles moving in opposite directions, and the mass ratio of the railway vehicles to the bridge (Hsu, 1996). A parametric study was carried out by Shen (1996) on a number of factors affecting the dynamic response of the bridge, in which both the modal superposition method and finite element method were employed. In the study by Wu *et al.* (2001), a bridge containing two railway tracks was considered, with which two trains are allowed to move over the bridge in opposite directions. Such a vehicle–rails–bridge interaction model was adopted by Wu (2000) and Yang and Wu (2002) in evaluating the risk of derailment for trains traveling over a bridge and simultaneously subjected to an earthquake excitation, and further by Wu and Yang (2003) in assessing the steady-state response and riding comfort of trains moving over a series of simply supported beams.

Based on an analytical approach, closed form solution has been obtained by Yang *et al.* (1997b) for the response of simple beams subjected to the passage of a high-speed train modeled as a sequence of moving loads with regular nonuniform intervals (Fig. 1.5), in which

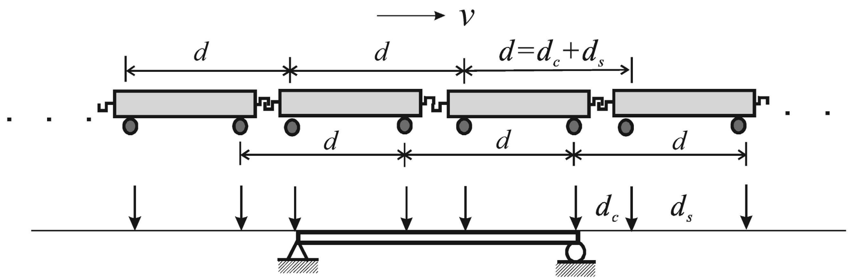


Fig. 1.5. Train load model.

the conditions for the phenomena of *resonance* and *cancellation* to occur have been identified. Based on these conditions, optimal design criteria that are effective for suppressing the resonant response of the VBI systems have been proposed. The same problem was later examined by Li and Su (1999) by using an alternative analytical approach, with similar findings obtained.

1.5. Methods of Solution

In studying the dynamic response of a VBI system, two sets of equations of motion can be written, one for the bridge and the other for the vehicles. It is the interaction or *contact forces* existing at the contact points of the two subsystems that make the two sets of equations coupled. As the contact points move from time to time, the system matrices are generally time-dependent, which must be updated and factorized at each time step. To solve these two sets of equations, procedures of an iterative nature have often been used (Hwang and Nowak, 1991; Green and Cebon, 1994; Yang and Fonder, 1996; Delgado and dos Santos, 1997). One way to do this is to start by assuming some values of displacements for the contact points, with which the contact forces can be solved from the vehicle equations. Next, by substituting the contact forces into the bridge equations, improved values of displacements for the contact points can be solved. The advantage of the iterative procedures is that the responses of both the vehicles and the bridge at any instant can be simultaneously made available. However, the convergence rate of iteration is likely to be low, when dealing with the more realistic case of bridges traveled by a large number of vehicles, whether in a random traffic flow, as encountered in highways, or in a connected line, as encountered in railways, for there exists twice the number of contact points if each vehicle is assumed to consist of two sets of wheel assemblies.

In the literature, Lagrange's equation with multipliers and constraint equations has also been applied to the analysis of VBI systems (Blejwas *et al.*, 1979). As it is well known, the use of Lagrange multipliers will increase the total number of unknowns and, therefore, the effort of computation. Theoretically speaking, a third approach is still possible, namely, by eliminating the interaction or contact forces from the original two sets of equations, one can form a new set of coupled equations for the entire VBI system. However, if the condensation procedure is performed on the structure level, the symmetry and other advantageous properties of the dynamic matrices associated with each subsystem will be destroyed (Yang and Lin, 1995).

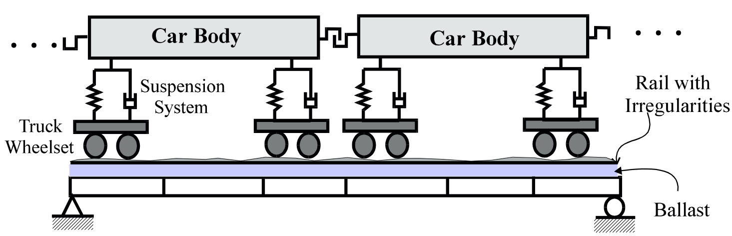
Perhaps, one of the most efficient approaches for solving the VBI equations is to perform the condensation technique on the element level. Garg and Dukkipati (1984) used the Guyan (1965) reduction scheme to condense the DOFs of the vehicles to those of the bridge. Recently, Yang and Lin (1995) used the *dynamic condensation method* to eliminate all the DOFs associated with each vehicle on the element level. Such approaches are good if only the response of the bridge (the larger subsystem) is desired. They may not yield accurate solutions for the response of the moving vehicles (the smaller subsystem), due to the approximations adopted in relating the vehicle (slave) DOFs to the bridge (master) DOFs.

Other methods that have been employed in solving the second-order differential equations of motion of the VBI problems include: (1) the direct integration methods, such as Newmark's β method (1959) (Inbanathan and Wieland, 1987; Yang and Lin, 1995), Wilson's θ method (Sridharan and Mallik, 1979), and fourth-order Runge–Kutta method (Chu *et al.*, 1986); (2) the modal superposition method (Blejwas *et al.*, 1979; Wu and Dai, 1987; Galdos *et al.*, 1993; Cai *et al.*, 1994), along with various integration schemes; and (3) the

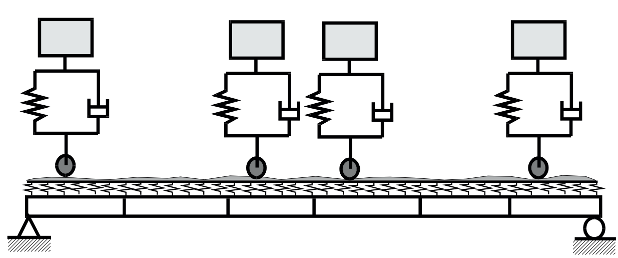
Fourier transformation method (Green and Cebon, 1994; Chang and Lee, 1994).

One essential feature with the VBI problem is that the number of vehicles acting on the bridge is time-dependent. The more the number of vehicles simultaneously acting on the bridge, the higher is the level for the vehicles to interact with the bridge. To overcome the dependency of the system matrices on the wheel load positions, i.e., the contact point positions, one feasible approach is to eliminate the DOFs of the vehicles not in direct contact with the bridge on the element level by the method of *dynamic condensation* (Yang and Lin, 1995). This will result in a VBI element that takes into account all the coupling effects. The following is a summary of the procedure presented by Yang and Yau (1997) for deriving the VBI element.

Consider a beam simulated by a number of elements traversed by a train, of which each railroad car is idealized as two lumped masses, each supported by a spring-dashpot unit, as shown in Fig. 1.6. For



(a)



(b)

Fig. 1.6. Train–bridge system: (a) general model and (b) sprung mass model.

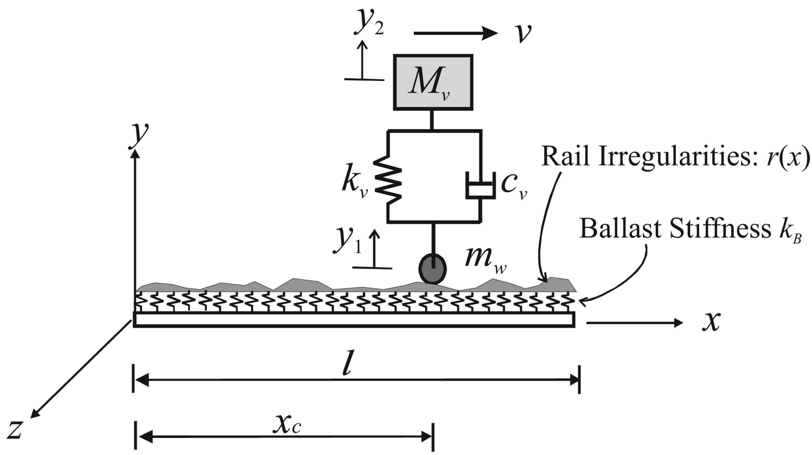


Fig. 1.7. Vehicle–bridge interaction element.

the present purposes, an *interaction element* is defined such that it consists of a beam element and a car-body mass and wheel mass connected by a suspension (spring-dashpot) unit directly acting over the beam element (Fig. 1.7). For the parts of the beam that are not directly under the action of the railroad cars, they can be modeled by conventional beam elements. However, for the remaining parts that are in direct contact with the wheel loads, the interaction elements have to be used instead.

With reference to the interaction element shown in Fig. 1.7, two sets of equations of motion can be written, one for the beam element and the other for the sprung mass unit. By Newmark's single-step finite difference formulas, the sprung mass equation can be discretized in time domain, from which the vehicle DOFs can be solved. Further, by the method of dynamic condensation, the sprung mass DOFs can be condensed to the associated DOFs of the beam element in contact. This will result in a VBI element with the effect of interaction fully taken into account. Since the VBI element possesses the same number of DOFs as the parent element, while retaining the properties of symmetry and bandedness in element matrices, it can be directly assembled with the conventional beam elements to form the structure equations. Such an element is particularly suitable for modeling bridges under a series of moving sprung masses with constant or varying intervals. It has the advantage that the response of the sprung mass can be recovered at any time step of the time-history analysis, which serves as a measure of the passengers' riding comfort. Using the VBI element, various dynamic properties of the beam and vehicles can be considered in the formulation, including the rail irregularities, ballast stiffness, damping of the beam, and stiffness and damping of the suspension units.

Recently, a more versatile approach for dealing with the VBI problems was proposed by Yang and Wu (2001). This method hinges on computation of the *contact forces* from the vehicles equations, in terms of the contact displacements. Before this can be done, the vehicle equations should first be discretized in time domain, say, using finite-difference equations of the Newmark type. The contact forces solved from the vehicles can then be treated as external loads and transformed as *consistent nodal loads* onto the bridge structure. With the bridge equations discretized in time domain, the bridge displacements can be solved as well. Such a procedure has been demonstrated to be quite flexible for treating vehicles of various complexities that appear in a sequence, in which both the vertical and horizontal contact forces are involved.

1.6. Impact Factor and Speed Parameter

In design practice, the dynamic response of a bridge has been considered indirectly by increasing the forces and stresses caused by the static live loads by an *impact factor*, defined as the ratio of the maximum dynamic to the maximum static response of the bridge under the same load minus one. One typical definition for the impact factor I is (Yang *et al.*, 1995):

$$I = \frac{R_d(x) - R_s(x)}{R_s(x)}\,, \tag{1.1}$$

where $R_d(x)$ and $R_s(x)$ denote respectively the maximum dynamic and static responses of the bridge calculated at the cross section x of the bridge of interest. The responses that may be considered for a beam include the deflection, bending moment, and shear force. The definition given in Eq. (1.1) is more rational and computationally more convenient than the *dynamic increment factor* (DIF) suggested by the AASHTO (*Guide*, 1980; Galdos *et al.*, 1993), or the *dynamic amplification factor* (DAF) discussed in Paultre *et al.* (1992) and Zhang *et al.* (2001b), since both the maximum dynamic and static responses are calculated at the same cross section of the bridge. Such an advantage will become obvious when dealing with moving loads that appear as a sequence, such as those of a train, or as a random flow, such as those encountered in highways, or in treating problems involving the resonance response, in the sense that the response of the bridge at the same cross section will be continuously amplified, as there are more loads passing the bridge. Care must be taken to distinguish the maximum impact factor from the maximum total response calculated for a beam. Occasionally, unreasonably large impact factors may be computed for a beam at some points due to the fact that the static responses, i.e., the denominator of Eq. (1.1), are very small. For this reason, the impact factor computed or measured for a bridge should not be regarded as the only criterion in the design of bridges.

It is well known that a number of factors may affect the impact factor of a bridge under the excitation of moving vehicular loads, for instance, the dynamic properties of the vehicle, the dynamic properties of the bridge, the vehicle speed, and the pavement roughness. Many *bridge codes*, including the American Association of State Highway and Transportation Officials (AASHTO) Specifications (*Standard*, 1989) and the Ontario Code (*Ontario*, 1983), have related the impact factor to a single parameter of the bridge, such as the span length or frequency of vibration, and have applied the same impact factor to all responses of the bridge including the deflection, shear force, and bending moment. According to the AASHTO Specifications, the impact factor I is related to the span length L of the bridge as

$$I = \frac{50}{L + 125} \leq 0.3 \tag{1.2}$$

when the bridge span length L is expressed in ft, and as

$$I = \frac{15.24}{L + 38.1} \leq 0.3 \tag{1.3}$$

when L is expressed in m. Formulas such as the preceding ones have been established several decades ago, based on limited field measurements, which are valid for the particular types of vehicles and bridges available in those days, if one realizes that modern trucks used are much heavier than those used half a century ago. The preceding formulas may be convenient for practical design, but are not theoretically sound at least for two reasons. First, the formulas are inconsistent in physical units, if one notes that the impact factor I itself is a nondimensional quantity, while the span length has some physical units. Second, the use of span length as the control parameter is not representative of the physical property of the bridge concerning the vehicle–bridge interactions. This is especially true for continuous beams, for which there exist more than one span lengths, none of which can be directly related to the modal vibration shape. Based on the evidence of more extensive theoretical analyses and field measurements, it was reported that impact factors calculated according to current design codes may significantly underestimate the bridge response in many cases (O'Connor and Pritchard, 1984; Inbanathan and Wieland, 1987; Galdos *et al.*, 1993; Huang *et al.*, 1993; Chang and Lee, 1994).

By denoting the velocity of the moving vehicle as v and the *characteristic length* of the beam as L, the exciting frequency of the moving vehicle can be expressed as $\pi v/L$. The *speed parameter* S that is particularly useful for expressing the dynamic response of the VBI system is defined as the ratio of the exciting frequency of the moving vehicle to the fundamental frequency ω of the beam, that is,

$$S = \frac{\pi v}{\omega L} \tag{1.4}$$

which is dimensionless. In the study by Yang *et al.* (1995), it was demonstrated that for VBI systems with a speed parameter S less than 0.5, the impact factor can be related to the speed parameter for the deflection, shear force, and bending moment of simple beams by linear formulas. Moreover, these formulas can be multiplied by some reduction coefficients to yield formulas for continuous beams. One particular point here is that the *characteristic length*, rather than the span length, should be used for the beam in defining the speed parameter. The former relates to the span length of a simple beam or the distance between two adjacent inflection points of the first mode of vibration of a continuous beam. It is with the use of the characteristic length in Eq. (1.4) that simple impact formulas can be established for both the simple and continuous beams (Yang *et al.*, 1997a). In the study by Pan and Li (2002), it was shown that the maximum displacement, velocity, and acceleration responses of the supporting structure appear to be almost linear to the speed parameter, which is consistent with the findings of Yang *et al.* (1995).

The speed parameter is an important parameter in the study of moving load problems, see, for instance, Tan and Shore (1968a,b), Veletsos and Huang (1970), Warburton (1976), Kurihara and Shimogo (1978b), Wang *et al.* (1992), Humar and Kashif (1993), Cai *et al.* (1994), and Chatterjee *et al.* (1994), among others. It was demonstrated that by plotting the response of the bridge against the speed parameter, rather than the span length or frequency of vibration that make up the parameter, generally more compact results can be obtained (Yang *et al.*, 1995). It was also noted by Paultre *et al.* (1992) that the DAF generally increases with the speed parameter. Noteworthy is the fact that the nondimensional character of the speed parameter enables us to extend the range of application of the computed or measured results to bridges beyond those covered in the study. Nevertheless, such a property was not fully recognized by a substantial portion of researchers working on vehicle-induced vibrations.

1.7. Concluding Remarks

A brief review of previous researches on the dynamic interaction of the bridge and moving vehicles was presented in this chapter. Vehicle models of increasing complexities, including the moving load, moving mass, sprung mass models, and more sophisticated ones, have been discussed. The factors that need to be considered in analyzing the response of the VBI systems include the dynamic properties and driving frequencies of the moving vehicles, and the dynamic properties and surface roughness of the bridge. Even though vehicle models of higher complexities, e.g., those consisting of dozens of DOFs, can be employed in studying the VBI problems nowadays, the use of simplified vehicle and bridge models is helpful, since it allows us to identify the key parameters dominating the dynamics of the VBI systems.

The impact factor adopted herein is computed based on quantities related to the same cross section of the beam that is of interest. It can be conveniently applied to cases involving a series of moving loads. The impact formulas provided by most current design codes are not consistent in physical units and lack a solid theoretical basis, of which the application should not be extended to bridges traveled by vehicles at high speeds. A more rational approach is to relate the impact factor, which is nondimensional, to the speed parameter, which is also nondimensional, defined as the ratio of the driving frequency of the moving vehicles to the vibration frequency of the bridge.

The VBI problem is a complicated one in that the contact points of the vehicles with the bridge move from time to time. Various methods exist for solving this problem. However, the most effective one appears to be the one based on condensation of the noncontact DOFs of the vehicle to the beam element in contact. The VBI element so derived can be applied to solving a great variety of VBI problems, by which the dynamic response of the moving vehicles, in addition to that for the bridge, can be obtained. Other factors that require further studies for high-speed railway bridges include the braking and acceleration of railroad cars, the torsional vibration of bridges caused by vehicles not moving along the centerline of the bridge girders, the crossing of two vehicles moving in opposite directions, the mass ratio of the vehicles to the bridge, the stability of rails, the risk of derailment of railroad cars under earthquake motions, and the stiffness effects of the ballast, elastic bearings, and supporting columns, among others.