

# Contents

<i>Preface</i>	vii
1. Introduction	1
1.1 Our aims . . . . .	1
1.2 Introducing <i>Maple</i> . . . . .	4
1.2.1 What is <i>Maple</i> ? . . . . .	4
1.2.2 Starting <i>Maple</i> . . . . .	4
1.2.3 Worksheets in <i>Maple</i> . . . . .	5
1.2.4 Entering commands into <i>Maple</i> . . . . .	6
1.2.5 Stopping <i>Maple</i> . . . . .	8
1.2.6 Using previous results . . . . .	8
1.2.7 Summary . . . . .	9
1.3 Help and error messages with <i>Maple</i> . . . . .	9
1.3.1 Help . . . . .	9
1.3.2 Error messages . . . . .	10
1.4 Arithmetic in <i>Maple</i> . . . . .	11
1.4.1 Basic mathematical operators . . . . .	11
1.4.2 Special mathematical constants . . . . .	12
1.4.3 Performing calculations . . . . .	12
1.4.4 Exact versus floating point numbers . . . . .	13
1.5 Algebra in <i>Maple</i> . . . . .	18
1.5.1 Assigning variables and giving names . . . . .	18
1.5.2 Useful inbuilt functions . . . . .	20
1.6 Examples of the use of <i>Maple</i> . . . . .	26
2. Sets	33

2.1	Sets . . . . .	33
2.1.1	Union, intersection and difference of sets . . . . .	35
2.1.2	Sets in <i>Maple</i> . . . . .	38
2.1.3	Families of sets . . . . .	42
2.1.4	Cartesian product of sets . . . . .	43
2.1.5	Some common sets . . . . .	43
2.2	Correct and incorrect reasoning . . . . .	45
2.3	Propositions and their combinations . . . . .	47
2.4	Indirect proof . . . . .	51
2.5	Comments and supplements . . . . .	53
2.5.1	Divisibility: An example of an axiomatic theory . . . . .	56
3.	Functions . . . . .	63
3.1	Relations . . . . .	63
3.2	Functions . . . . .	68
3.3	Functions in <i>Maple</i> . . . . .	75
3.3.1	Library of functions . . . . .	75
3.3.2	Defining functions in <i>Maple</i> . . . . .	77
3.3.3	Boolean functions . . . . .	80
3.3.4	Graphs of functions in <i>Maple</i> . . . . .	81
3.4	Composition of functions . . . . .	89
3.5	Bijections . . . . .	90
3.6	Inverse functions . . . . .	91
3.7	Comments . . . . .	95
4.	Real Numbers . . . . .	97
4.1	Fields . . . . .	97
4.2	Order axioms . . . . .	100
4.3	Absolute value . . . . .	105
4.4	Using <i>Maple</i> for solving inequalities . . . . .	108
4.5	Inductive sets . . . . .	112
4.6	The least upper bound axiom . . . . .	114
4.7	Operation with real valued functions . . . . .	120
4.8	Supplement. Peano axioms. Dedekind cuts . . . . .	121
5.	Mathematical Induction . . . . .	129
5.1	Inductive reasoning . . . . .	129
5.2	Aim high! . . . . .	135

5.3	Notation for sums and products . . . . .	136
5.3.1	Sums in <i>Maple</i> . . . . .	141
5.3.2	Products in <i>Maple</i> . . . . .	143
5.4	Sequences . . . . .	145
5.5	Inductive definitions . . . . .	146
5.6	The binomial theorem . . . . .	149
5.7	Roots and powers with rational exponents . . . . .	152
5.8	Some important inequalities . . . . .	157
5.9	Complete induction . . . . .	161
5.10	Proof of the recursion theorem . . . . .	164
5.11	Comments . . . . .	166
6.	Polynomials . . . . .	167
6.1	Polynomial functions . . . . .	167
6.2	Algebraic viewpoint . . . . .	169
6.3	Long division algorithm . . . . .	175
6.4	Roots of polynomials . . . . .	178
6.5	The Taylor polynomial . . . . .	180
6.6	Factorization . . . . .	184
7.	Complex Numbers . . . . .	191
7.1	Field extensions . . . . .	191
7.2	Complex numbers . . . . .	194
7.2.1	Absolute value of a complex number . . . . .	196
7.2.2	Square root of a complex number . . . . .	197
7.2.3	Maple and complex numbers . . . . .	199
7.2.4	Geometric representation of complex numbers. Trigonometric form of a complex number . . . . .	199
7.2.5	The binomial equation . . . . .	202
8.	Solving Equations . . . . .	207
8.1	General remarks . . . . .	207
8.2	<i>Maple</i> commands <code>solve</code> and <code>fsolve</code> . . . . .	209
8.3	Algebraic equations . . . . .	213
8.3.1	Equations of higher orders and <code>fsolve</code> . . . . .	225
8.4	Linear equations in several unknowns . . . . .	228
9.	Sets Revisited . . . . .	231

9.1	Equivalent sets . . . . .	231
10.	Limits of Sequences	239
10.1	The concept of a limit . . . . .	239
10.2	Basic theorems . . . . .	249
10.3	Limits of sequences in <i>Maple</i> . . . . .	255
10.4	Monotonic sequences . . . . .	257
10.5	Infinite limits . . . . .	263
10.6	Subsequences . . . . .	267
10.7	Existence theorems . . . . .	268
10.8	Comments and supplements . . . . .	275
11.	Series	279
11.1	Definition of convergence . . . . .	279
11.2	Basic theorems . . . . .	285
11.3	<i>Maple</i> and infinite series . . . . .	289
11.4	Absolute and conditional convergence . . . . .	290
11.5	Rearrangements . . . . .	295
11.6	Convergence tests . . . . .	297
11.7	Power series . . . . .	300
11.8	Comments and supplements . . . . .	303
	11.8.1 More convergence tests. . . . .	303
	11.8.2 Rearrangements revisited. . . . .	306
	11.8.3 Multiplication of series. . . . .	307
	11.8.4 Concluding comments . . . . .	309
12.	Limits and Continuity of Functions	313
12.1	Limits . . . . .	313
	12.1.1 Limits of functions in <i>Maple</i> . . . . .	319
12.2	The Cauchy definition . . . . .	322
12.3	Infinite limits . . . . .	329
12.4	Continuity at a point . . . . .	332
12.5	Continuity of functions on closed bounded intervals . . . . .	338
12.6	Comments and supplements . . . . .	353
13.	Derivatives	357
13.1	Introduction . . . . .	357
13.2	Basic theorems on derivatives . . . . .	362

13.3	Significance of the sign of derivative. . . . .	369
13.4	Higher derivatives . . . . .	380
13.4.1	Higher derivatives in <i>Maple</i> . . . . .	381
13.4.2	Significance of the second derivative . . . . .	382
13.5	Mean value theorems . . . . .	388
13.6	The Bernoulli–l’Hospital rule . . . . .	391
13.7	Taylor’s formula . . . . .	394
13.8	Differentiation of power series . . . . .	398
13.9	Comments and supplements . . . . .	401
14.	Elementary Functions . . . . .	407
14.1	Introduction . . . . .	407
14.2	The exponential function . . . . .	408
14.3	The logarithm . . . . .	411
14.4	The general power . . . . .	415
14.5	Trigonometric functions . . . . .	418
14.6	Inverses to trigonometric functions. . . . .	425
14.7	Hyperbolic functions . . . . .	430
15.	Integrals . . . . .	431
15.1	Intuitive description of the integral . . . . .	431
15.2	The definition of the integral . . . . .	438
15.2.1	Integration in <i>Maple</i> . . . . .	443
15.3	Basic theorems . . . . .	445
15.4	Bolzano–Cauchy principle . . . . .	450
15.5	Antiderivates and areas . . . . .	455
15.6	Introduction to the fundamental theorem of calculus . . . . .	457
15.7	The fundamental theorem of calculus . . . . .	458
15.8	Consequences of the fundamental theorem . . . . .	469
15.9	Remainder in the Taylor formula . . . . .	477
15.10	The indefinite integral . . . . .	481
15.11	Integrals over unbounded intervals . . . . .	488
15.12	Interchange of limit and integration . . . . .	492
15.13	Comments and supplements . . . . .	498
Appendix A	<i>Maple</i> Programming . . . . .	501
A.1	Some <i>Maple</i> programs . . . . .	501
A.1.1	Introduction . . . . .	501

A.1.2	The conditional statement . . . . .	502
A.1.3	The <b>while</b> statement . . . . .	504
A.2	Examples . . . . .	506
<i>References</i>		511
<i>Index of Maple commands used in this book</i>		513
<i>Index</i>		519