PREFACE

The subject of the spontaneous formation of spatio-temporal structures in systems far from equilibrium, generally referred to as patterns, is an exciting and fast-growing branch of applied mathematics and physics, with strong impact on fields as diverse as ecology, chemistry, engineering, social sciences, as well as new technologies and processes. Systems with different microscopic descriptions frequently exhibit similar patterns on a macroscopic level. The emergence of macroscopic order and the spontaneous breaking of spatiotemporal symmetries are features common to most natural phenomena, and many structures seen in our world can be considered the result of a sequence of successive symmetry-breaking instabilities caused by nonlinear processes under non-equilibrium conditions.

Although studied intensively for most of the last century, it has only been during the past thirty years that pattern formation has emerged as an own branch of science. Understanding the spontaneous formation and dynamics of spatio-temporal patterns in dissipative non-equilibrium systems is one of the major challenges in nonlinear science, and it is to be expected that new concepts and methods resulting from this field will influence future developments in many disciplines. Recent experimental results have demonstrated a variety of new patterns that can be observed in macroscopic as well as microscopic systems far from equilibrium, which are mostly unexplored and demand a theoretical description.

Studies of pattern formation and pattern dynamics use a common set of fundamental concepts to describe how non-equilibrium processes cause structures to appear in a wide range of systems in nature and technology. A good step towards the description of patterns and the fundamental understanding of nonlinear processes in the large is provided by studying dissipative systems, in which energy is supplied through a gradient such as in temperature, velocity, concentration etc. that maintains the system away from equilibrium.

Mathematical Approaches. Experimental and theoretical investigations of patterns are aimed at understanding the mechanisms involved in the formation and the selection of patterns, as well as their stability, temporal evolution and control. Addressing these issues mathematically requires a wide range of techniques involving ideas from dynamical systems, functional analysis, group theory, perturbation methods, ordinary and partial differential equations, geometry and singularity theory, as well as the ability to find the appropriate mathematical framework that adequately describes an experimentally observed phenomenon.

Equivariant Bifurcation Theory. Since the seminal paper of Turing appeared in 1952, it is well known that pattern formation is intimately connected with instabilities and spontaneously broken symmetries. This observation has led to the development of equivariant bifurcation theory, initiated by Sattinger in the 1970's and established as a branch of mathematics by Golubitsky and Stewart in the 1980's. In equivariant bifurcation theory the patterns studied are usually spatially or spatio-temporally periodic. This restriction allows to reduce an extended system to a finite dimensional system of "normal forms" near an instability, and the application of equivariant bifurcation theorems allows to predict the patterns that can be expected above threshold and to study their stability against periodic perturbations. Since the discretization of spatial systems usually leads to finite but high dimensional systems of equations, the numerical continuation of these patterns and the detection of secondary bifurcations require efficient algorithms which are a challenge for numerical analysts.

Modulation Approach. On the other hand, the modulation or envelope approach, also referred to as "Ginzburg Landau formalism", takes spatial modulations of periodic patterns into account, which are governed by space and time dependent amplitude or envelope equations. This approach was initiated by Newell, Whitehead and Segal in the late 1960's in the context of fluid mechanics. Many of the qualitative and quantitative theoretical predictions based on the amplitude equation approach have been successfully confronted with experiments, but a rigorous mathematical justification was lacking until the 1990's. In the meantime a number of theorems that justify this approach rigorously are available, however all of these results are for autonomous systems and so exclude time periodic systems such as the equations describing the Farady experiment or ac–driven electroconvection in liquid crystals.

Cellular Automata and Lattice Gases. The above approaches to study pattern formation and pattern dynamics rely on a macro- or mesoscopic, i.e.

deterministic level of description of the physical system through a governing set of nonlinear partial differential equations. For most of the systems studied in pattern formation, a fully microscopic description on the basis of statistical physics or molecular dynamics is untractable. Simplified models designed to understand the spatio-temporal evolution of macroscopic systems on a microscopic level are provided by cellular automata and lattice gases, which date back to von Neumann in the 1930's. Triggered by the availability of high power computing systems, there has been growing interest in simulations of these models to study collective phenomena. Cellular automata and lattice gases show a rich variety of patterns already for very simple rules of evolution.

Complex Patterns. In the past fifteen years we have witnessed significant progress in the field, and the focus shifted to more and more complex patterns. Progress in equivariant bifurcation theory encompasses superlattice patterns and quasi-patterns, and the extension to non-compact Euclidean symmetry resulted in a solid mathematical theory for the bifurcation of spiral and target waves. Studies of amplitude and phase diffusion equations, in particular the complex Ginzburg Landau equation, have led to a good qualitative understanding of different types of weak turbulence and transitions between them, see Chapter 8 for a survey. These phenomena are intrinsically spatio-temporal, and so cannot be described by finite dimensional dynamical systems. On the other hand, the observation of heteroclinic cycles as robust phenomena in finite dimensional systems with symmetry triggered much research in equivariant dynamics. Various new types of intermittency have been described and put on a solid mathematical basis.

While much progress towards understanding pattern formation and dynamics has been made in recent years, fundamental challenges remain. The basic question of whether universality classes exist for patterning behavior is still unanswered, and the characterization of patterns that are complex in both space and time (spatiotemporal chaos) is far from being complete. Observation of localized structures confined to a small spatial region of the system (e.g. "oscillons" and "worms") have also led to additional questions, for example why such states do not expand to fill the entire domain. Progress towards resolving fundamental questions of pattern formation also has significant practical implications for control since many technological processes involve pattern formation at some stage.

Contents of the Book. The present book emerged from a workshop on *Dynamics and Bifurcation of Patterns in Dissipative Systems* organized by the editors in May 2003 at Colorado State University. The book focuses on key ideas, new advances and open questions in the description and analysis of spatiotemporal patterns in dissipative extended systems. In a collection of expository papers and advanced research articles, written by leading experts ranging from applied mathematicians to theoretical and experimental physicists, the book provides an overview of the current state of the art in dynamics and bifurcation of patterns. The topics include mathematical issues related to bifurcations and instabilities in spatio-temporally continuous systems and the role of symmetry, advances in the study of localized patterns, dissipative waves, and weak turbulence, and new approaches to the modeling and characterization of spatio-temporal complexity. The applications encompass a remarkable variety of applied fields such as magneto- and binary fluid convection, liquid crystals, granular media, Faraday waves, multiscale biological patterns, visual hallucinations, and biological pacemakers.

The book is divided into three major parts, in which contributions using common mathematical methods or addressing related mathematical questions are grouped together. Each part begins with a relatively broad survey on recent research results, followed by contributions addressing more specific questions.

Part I is dedicated to **instabilities, bifurcation and the role of symmetry**. Chapter 1 gives a survey on recent and previous results on pattern formation in the visual system, and shows how these results lead to a classification of visual hallucinations. In Chapter 2 the authors discuss new efficient numerical algorithms for the continuation of periodic solutions of high dimensional systems, the detection of bifurcations, and the analysis of instabilities, including an application to electroconvection in nematic liquid crystals. Chapter 3 makes a first step towards the rigorous justification of the Ginzburg Landau formalism for time periodically driven systems. Chapter 4 discusses the stability and bifurcation of families of equilibria and periodic orbits due to continuous symmetries (referred to as relative equilibria and relative periodic orbits), which are important for analyzing bifurcations of spiral and target waves. In Chapter 5 the problem of rotating magnetoconvection with magnetostrophic balance, originally formulated by Chandrasekhar, is studied. The authors notice that Chandrasekhar's discussion is incomplete and find a complex sequence of transitions involving three oscillatory modes as parameters are varied in certain regimes of astrophysical interest. Chapter 6 reports on new results on pattern formation on a sphere for the even l mode representations of the spherical symmetry group $O(3)$. Chapter 6 discusses the convergence properties of Fourier mode representations of quasipatterns and the problems encountered by virtue of the presence of small divisors in a perturbation analysis.

The subject of **Part II** are **localized patterns, waves, and weak turbulence**. The part begins in Chapter 8 with a survey on phase diffusion, phase instabilities, and weak turbulence as described by classical envelope equations, in particular the complex Ginzburg Landau equation, that also includes a brief review of statistical approaches in the description of patterns. Chapter 9 discusses the merger of pattern formation and parametric resonance in terms of a new prototype system, the Mathieu partial differential equation, aiming at a description of localized patterns observed in the Faraday experiment and in shaked granular media. Averaging leads here to a dissipatively perturbed nonlinear Schrödinger equation. Related to Chapter 9 is Chapter 10, in which mean flow effects in model equations for the Faraday waves are analyzed that lead to new types of instabilities. In Chapter 11 rogue waves and the Benjamin-Feir instability are studied in the framework of a dispersively perturbed nonlinear Schrödinger equation. Here a Melnikov analysis shows that rogue waves are well approximated by homoclinic solutions of the unperturbed equation. The subject of Chapter 12 are target patterns produced by heterogeneous pacemakers in oscillatory media described by reaction diffusion equations with space dependent oscillation frequency.

Part III deals with the **modelling and characterization of spatio-temporal complexity**. The first chapter, Chapter 13, gives a survey on recent results in which bursting behaviour observed in different fluid systems has been successfully described by a common finite dimensional mechanism. Related to Chapter 13 are Chapters 18 and 19. In Chapter 18 the destruction of symmetry-enforced, robust heteroclinic cycles through symmetry breaking imperfections and the resulting complex dynamics are studied, whereas in Chapter 19 the new concept of internal dynamics of intermittency is introduced and illustrated by examples. Chapters 15–17 are devoted to the characterization of complex spatio-temporal behavior by extracting certain characteristics from computed or measured data. The subject of Chapter 15 is the computation of coherent structures and their temporal evolution from simulated hurricane data. The authors apply the classical Karhunen Loeve projection and the newly introduced signal fraction analysis projection and compare the results obtained with these two methods. In Chapter 16 the characteristics are based on an extension of the Nusselt number used in Rayleigh Benard convection to ac-driven electroconvection in nematic liquid crystals, which leads here to a distinction in linear and higher order nonlinear Nusselt numbers. In Chapter 17 the extraction of characteristics of far from equilibrium structures using their contours is described and two measures are discussed that can be used to describe labyrinthine patterns and growth interfaces. Chapters 14 and 20 discuss pattern formation and morphogenesis in developmental biology from a modeling and experimental perspective, respectively. In Chapter 14 several stochastic lattice gas models capable of describing different biological systems are reviewed, and in Chapter 20 the formation of aggregation mounds of a certain amoebae species under different geometrical constraints is studied.

The unitary presentations in the book, guiding the reader from basic fundamental concepts to most recent research results, makes it suitable for a wide group of graduate students and postgraduates of applied mathematics and theoretical physics, as well as any researcher interested in pattern formation and nonlinear instabilities.

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