

## Chapter 2

# Exotics

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We review lattice QCD results for glueballs (including a discussion of mixing with scalar mesons), hybrid mesons and other exotic states (such as  $B_s B_s$  molecules).

### 2.1 Introduction

Quantum Chromodynamics has emerged as the unique theory to describe hadronic physics. It is formulated in terms of gluonic and quark fields. The only free parameters are the scale of the coupling (usually called  $\Lambda_{QCD}$ ) and the quark masses defined at some conventional energy scale.

Where large momentum transfers occur, the effective coupling becomes weak and a perturbative treatment is valid: in this domain the theory has been tested directly by experiment. However, because the effective coupling is weak for these processes that can be described by perturbation theory, they are necessarily not the dominant hadronic processes. A typical hadronic process will involve small momentum transfers and so has to be treated non-perturbatively.

In this non-perturbative régime, the description of hadrons is quite far removed from the description of the gluonic and quark fields in the QCD Lagrangian. Because only colour-singlet states survive, the hadrons are all composites of quarks and gluons. One example emphasises this: the nucleon has a mass which is very much greater than the sum of the quark masses of the three valence quarks comprising it. This extra mass comes from the gluonic interactions of QCD. Another way to view this is that the naïve

Table 2.1 Non-exotic spin-parity combinations  $J^{PC}$  for both singlets ( $S = 0$ ) and triplets ( $S = 1$ ), where  $J = L + S$ .

$L$	$S = 0$	$S = 1$		
0	$0^{-+}$	$1^{--}$		
1	$1^{+-}$	$0^{++}$	$1^{++}$	$2^{++}$
2	$2^{-+}$	$1^{--}$	$2^{--}$	$3^{--}$

quark model is a useful phenomenological tool but has constituent quarks with masses much greater than the QCD masses (*i.e.* masses as defined in the Lagrangian). It is important to understand why this is approximately what QCD requires and to find where QCD departs from the naïve quark model.

One way to characterise the manner in which QCD goes beyond the naïve quark model is through the concept of exotic states. Here exotic is taken to mean ‘not included in the naïve quark model’. In order to discuss exotic states, we need to summarise what the naïve quark model contains. Basically the degrees of freedom are the valence quarks (*i.e.* quark-antiquark for a meson and 3 quarks for a baryon) with masses and interactions given by some effective interaction. The consequences of this are that only certain  $J^{PC}$  values will exist and that the number of states with different quark flavours is specified. So, concentrating on mesons made of the three flavours of light quarks ( $u$ ,  $d$ ,  $s$ ), one expects a nonet of mesons with the flavours ( $\bar{u}d$ ,  $\bar{d}u$ ,  $\bar{u}u \pm \bar{d}d$ ,  $\bar{s}s$ ,  $\bar{u}s$ ,  $\bar{d}s$ ,  $\bar{s}u$ ,  $\bar{s}d$ ). This is indeed what is found for vector mesons ( $\rho$ ,  $\omega$ ,  $\phi$ ,  $K^*$ ). It is also possible within the quark model for the flavour-singlet states ( $\bar{u}u + \bar{d}d$ ,  $\bar{s}s$ ) to mix, as found for the pseudoscalar mesons. What would be exotic is for a tenth state to exist. For mesons with orbital angular momentum  $L$  between the quark and antiquark the allowed  $J^{PC}$  values are shown in Table 2.1. Thus spin-parity combinations such as  $0^{--}$ ,  $0^{+-}$ ,  $1^{-+}$ ,  $2^{+-}$  are termed spin-exotic since they cannot be made from a quark plus antiquark alone. It has been a considerable challenge to build a machinery that allows non-perturbative calculations in QCD with all systematic errors determined. The most controlled approach to non-perturbative QCD is via lattice techniques in which space-time is discretized and time is taken as Euclidean. The functional integral is then

evaluated numerically using Monte Carlo techniques.

Lattice QCD needs as input the quark masses and an overall scale (conventionally given by  $\Lambda_{\text{QCD}}$ ). Then any Green function can be evaluated by taking an average of suitable combinations of the lattice fields in the vacuum samples. This allows masses to be studied easily and matrix elements (particularly those of weak or electromagnetic currents) can be extracted straightforwardly. Unlike experiment, lattice QCD can vary the quark masses and can also explore different boundary conditions and sources. This allows a wide range of studies which can be used to diagnose the health of phenomenological models as well as casting light on experimental data.

One limitation of the lattice approach to QCD is in exploring hadronic decays because the lattice, using Euclidean time, has no concept of asymptotic states. One feasible strategy is to evaluate the mixing between states of the same energy — so giving some information on on-shell hadronic decay amplitudes.

There is an interesting theoretical world in which the quark degrees of freedom are removed from QCD, leaving pure gluo-dynamics. This is also known as pure Yang-Mills theory. It is a self-consistent theory which has the full non-perturbative gluonic interaction. It turns out that this gluonic interaction does produce the salient features of QCD: asymptotic freedom, confinement, *etc.* It is of interest to explore the spectrum in this case: the states are called glueballs.

It is also of interest to consider the propagation of quarks in this gluonic theory. The quarks are treated as in the Dirac equation and they propagate through the gluonic ground state. This approach is known as the quenched approximation. Again this turns out to be a very useful approximation: chiral symmetry breaking occurs for example. This quenched approximation is in contrast to the full quantum field theory of QCD where there would be quark loop effects in the ground state also. Thus in the quenched approximation there will be inconsistencies: the theory is not unitary. However, for heavy quarks it will be a good approximation since heavy quark loops are suppressed and it may be adequate to describe some features of lighter quarks. Moreover, many phenomenological models are appropriate to the quenched case and so can be compared with quenched QCD.

## 2.2 Glueballs and Scalar Mesons

### 2.2.1 Glueballs in quenched QCD

Glueballs are defined to be hadronic states made primarily from gluons. The full non-perturbative gluonic interaction is included in quenched QCD. A study of the glueball spectrum in quenched QCD is thus of great value. This will allow experimental searches to be guided as well as providing calibration for models of glueballs. A non-zero glueball mass in quenched QCD is the “mass-gap” of QCD. To prove this rigorously is one of the major challenges of our times. Here we will explore the situation using computational techniques.

In lattice studies, dimensionless ratios of quantities are obtained. To explore the glueball masses  $m$ , it is appropriate to combine them with another very accurately measured quantity to have a dimensionless observable. Since the potential between static quarks is very accurately measured from the lattice, it is now conventional [1] to use  $r_0$  for this comparison. Here  $r_0$  is implicitly defined by  $r^2 dV(r)/dr = 1.65$  at  $r = r_0$  where  $V(r)$  is the potential energy between static quarks which is easy to determine accurately on the lattice. Conventionally  $r_0 \approx 0.5$  fm.

Theoretical analysis indicates that for Wilson’s discretisation of the gauge fields in the quenched approximation, the dimensionless ratio  $mr_0$  will differ from the continuum limit value by corrections of order  $a^2$ . Thus in Fig. 2.1 the mass of the  $J^{PC}=0^{++}$  glueball is plotted versus the lattice spacing  $a^2$ . The straight line then shows the continuum limit obtained by extrapolating to  $a = 0$ . As can be seen, there is essentially no need for data at even smaller  $a$ -values to further fix the continuum value. The value shown corresponds to  $m(0^{++})r_0 = 4.33(5)$ . Since several lattice groups [2, 3, 4, 5] have measured these quantities, it is reassuring to see that the purely lattice observables are in excellent agreement. The publicised difference of quoted  $m(0^{++})$  from UKQCD [4] and GF11 [5] comes entirely from relating quenched lattice measurements to values in GeV.

In the quenched approximation, different hadronic observables differ from experiment by factors of up to 10%. Thus using one quantity or another to set the scale, gives an overall systematic error. Here the scale is set by taking the conventional value of the string tension (determined from potential models and from hadronic lattice studies),  $\sqrt{\sigma} = 0.44$  GeV, which then corresponds to  $r_0^{-1} = 373$  MeV (or  $r_0 = 0.53$  fm). An over-

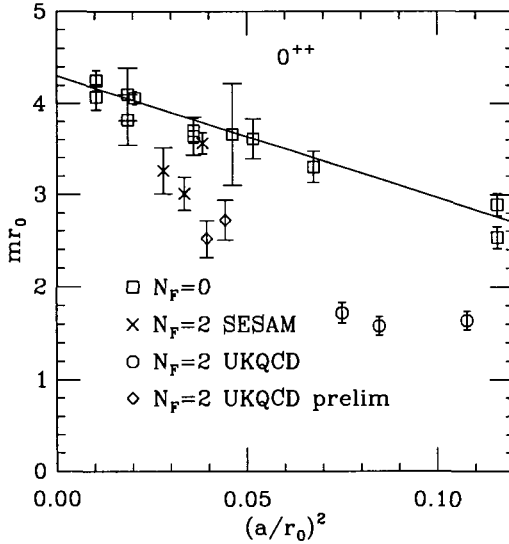


Fig. 2.1 The value of the mass of the  $J^{PC} = 0^{++}$  glueball state from quenched data ( $N_F = 0$ ) [2, 3, 4, 5] in units of  $r_0$  where  $r_0 \approx 0.5$  fm. The straight line shows a fit describing the approach to the continuum limit as  $a \rightarrow 0$ . Results [6, 7, 8] for the lightest scalar meson with  $N_F = 2$  flavours of sea quarks are also shown.

all systematic error of 10% is then to be included to any extracted mass. This yields  $m(0^{++}) = 1611(30)(160)$  MeV where the second error is the systematic scale error. Note that this is the glueball mass in the quenched approximation — in the real world significant mixing with  $q\bar{q}$  states *etc.* may modify this value substantially, as we discuss below.

In the Wilson approach, the next lightest glueballs are [3, 4] the tensor  $m(2^{++})r_0 = 6.0(6)$  [resulting in  $m(2^{++}) = 2232(220)(220)$  MeV] and the pseudoscalar  $m(0^{-+})r_0 = 6.0(1.0)$ . Although the Wilson discretisation provides a definitive study of the lightest  $(0^{++})$  glueball in the continuum limit, other methods are competitive for the determination of the mass of heavier glueballs. Namely, using an improved gauge discretisation which has even smaller discretisation errors than the  $a^2$  dependence of the Wilson discretisation, so allowing a relatively coarse lattice spacing  $a$  to be used. To extract mass values, one has to explore the time dependence of correlators and for this reason, it is optimum to use a relatively small time lattice spacing. Thus an asymmetric lattice spacing is most appropriate. The results [9] are shown in Fig. 2.2 and for low lying states are that

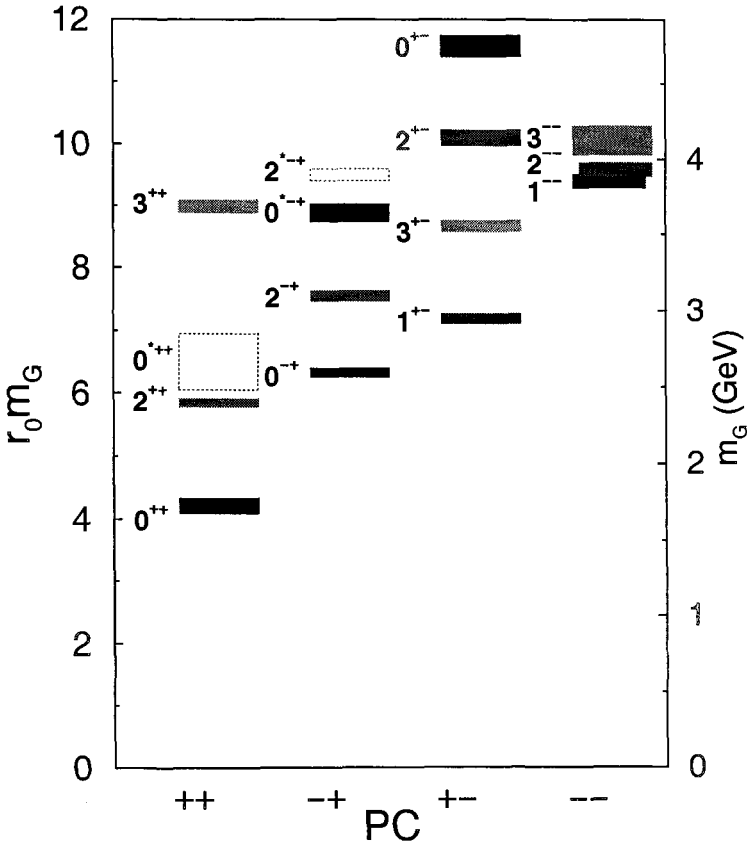


Fig. 2.2 The continuum glueball spectrum[9].

$m(0^{++})r_0 = 4.21(11)(4)$ ,  $m(2^{++})r_0 = 5.85(2)(6)$ ,  $m(0^{-+})r_0 = 6.33(7)(6)$  and  $m(1^{+-})r_0 = 7.18(4)(7)$ . It will be very difficult to identify experimentally states corresponding to these heavier glueballs since the spectrum is rich in  $q\bar{q}$  states of those quantum numbers at those mass values and there will thus be considerable mixing.

One signal of great interest would be a glueball with  $J^{PC}$  not allowed for  $q\bar{q}$  — a spin-exotic glueball or *oddball* — since it would not mix with  $q\bar{q}$  states. These states are found [3, 4, 9] to be high lying: considerably above  $2m(0^{++})$ . Thus they are likely to be in a region around 4 GeV where it is very difficult to separate states unambiguously by experiment.

As well as the mass of a glueball, it is possible to study their physical size. In principle this is determined by measuring the matrix element  $\langle G|J|G\rangle$  where  $J$  is some local current which couples to the glueball. Since glueballs have no flavour, the energy-momentum tensor is the most appropriate choice. A preliminary study has been made, albeit with large systematic errors [10] and finds a radius  $0.9 \pm 0.3$  fm. This approach should be contrasted with what has come to be called the Bethe-Salpeter wavefunction which is obtained from  $\langle G|L|0\rangle$  where  $L$  is the lattice operator used to create a glueball state from the vacuum. Within a lattice calculation, it is easy to measure the dependence of this overlap on the spatial extent of  $L$  [11], but difficult to interpret the result.

Related information can be obtained from a study of the gluelump: the state with one static colour source in the octet representation with a gluonic field making it a colour singlet. This would be of physical relevance should a massive gluino exist: it would be the glueballino, a gluino-gluon bound state. The spectrum [12] and spatial distribution [13] have been studied.

Another topic which is of mainly theoretical interest is the glueball mass (as a dimensionless ratio to the string tension) as the number of colours  $N_C$  is varied from 3. The SU(2) Yang-Mills theory has often been studied, especially as it is computationally simpler. Recently a study of SU( $N_C$ ) for  $N_C=4$  and 5 has been made. The summary [14] is that  $N_C = \infty$  is relatively close to  $N_C=3$ . This has theoretical implications since the  $N_C = \infty$  theory is formally simpler. For a comparison of these lattice results with ADS supergravity see Ref. [15].

### 2.2.2 Scalar mesons in quenched QCD

In quenched QCD the flavour singlet ( $f_0$ ) and non-singlet ( $a_0$ ) scalar mesons are degenerate. In full QCD this degeneracy is split by disconnected quark diagrams but these are omitted from the quenched approximation. This same feature of the quenched approximation implies that the  $\eta$  meson is wrongly treated - it will be degenerate with the  $\pi$ . This implies that the scalar meson propagation can have the wrong sign [16] because the  $\eta\pi$  intermediate state is mistreated (once quark loops are allowed in the vacuum then this anomaly is removed). For light quarks of mass corresponding to the strange quark or heavier, it is expected that this anomaly is relatively unimportant. Thus the measurement of the mass of the  $q\bar{q}$  scalar meson can be particularly unreliable in the quenched approximation.

Even though the mixing of the glueball and  $q\bar{q}$  states is not implemented in the quenched approximation, one can determine the mixing matrix element. This can then be used to estimate the result of the mixing by hand (by using a mass matrix for example). On a rather coarse lattice, where  $a^{-1} \approx 1.2$  GeV, two groups have attempted to measure this mixing [7, 17]. Their results expressed as the mixing for two degenerate quarks of mass around the strange quark mass are similar, namely  $E \approx 0.3$  GeV [17] and 0.5 GeV [7]. This is a relatively large mixing (if the glueball and scalar meson states were degenerate they would be split by  $\pm E$ ).

An exploratory attempt to extrapolate this mixing to the continuum [17] gave a very small mixing of 61(45) MeV, while the other determination [7] uses clover improvement so order  $a$  effects in the extrapolation to the continuum are suppressed and one would not expect a significant decrease in going to the continuum limit. What this discussion shows is that precision studies of the mixing on a quenched lattice have not yet been achieved. Furthermore the problems with the scalar meson propagation in the quenched approximation discussed above also limit progress.

As well as this mixing of the glueball with  $q\bar{q}$  states, there will be mixing with  $q\bar{q}q\bar{q}$  states which will be responsible for the hadronic decays. A first attempt to study this [18] at a coarse lattice spacing yields an estimated width for decay to two pseudoscalar mesons from the scalar glueball of order 100 MeV. A more realistic study would involve taking account of mixing with the  $n\bar{n}$  and  $s\bar{s}$  scalar mesons as well.

### 2.2.3 Scalar mesons in full QCD

It is now feasible to explore the flavour-singlet scalar meson spectrum including the quark loops in the vacuum, *i.e.* in full QCD. From dynamical fermion studies with  $N_f = 2$ , one can determine the flavour singlet and non-singlet mass spectrum. What is found [7, 8] is that the lightest flavour-singlet scalar meson ( $f_0$ ) is lighter than the lightest flavour non-singlet ( $a_0$ ).

The interpretation of this study is hampered by the same issue that hampers the interpretation of experimental data, namely, the mass eigenstates are not distinguished as 'glueball' or as 'quark-antiquark'. What one can do is explore the output spectrum and deduce what mixing might have occurred. To give an example, where we restrict here to  $N_f = 2$  flavours of degenerate quarks, the  $f_0$  masses will be  $m_0$  and  $m_0'$  where the latter is the first excited state and the flavour non-singlet  $a_0$  mass will be  $m_1$ .

Results for  $m_0$  are given in Fig. 2.1. Then one would expect in a simple  $2 \times 2$  mixing scenario (*i.e.* glueball and  $q\bar{q}$  meson) a mass matrix

$$\begin{pmatrix} m_G & E \\ E & m_1 \end{pmatrix},$$

where  $m_G$  is the glueball mass and  $E$  the mixing matrix element. This will have two mass eigenstates which can be identified with  $m_0$  and  $m_0'$  so determining the two free parameters in the matrix. This approach explains what is going on but obtains two numbers with two parameters, so there is no cross-check.

One can directly address the issue of the mass of the lightest scalar singlet meson from the lattice with  $N_f = 2$ . It is advantageous to use as full a basis of lattice operators as possible, including Wilson loops and quark-antiquark loops. Including the latter can in practice lead to a lower value of the ground state scalar meson mass - see Refs. [20, 8]. Most studies have shown no significant change of the scalar glueball mass as dynamical quarks are included [6]. However the larger lattice spacing result [7] shows a significant reduction in the lightest scalar mass, as shown in Fig. 2.1. Before concluding that this implies a lower scalar mass in the continuum limit, one needs to check whether an enhanced order  $a^2$  correction might be present. The origin of the large coefficient of  $a^2$  in quenched glueball studies is usually ascribed to the presence of a critical point in the fundamental-adjoint coupling plane which is close by in the usual Wilson approach with zero adjoint coupling. The extent to which this will be enhanced/reduced when dynamical quarks are introduced is not clear. Studies using the same approach at a finer lattice spacing [8, 20] do suggest that this large order  $a^2$  effect is significant for dynamical quarks, but studies even nearer to the continuum or with improved actions are needed to resolve this fully.

A further complication is that as the quark mass is reduced towards the physical light quark mass, the decay to  $\pi\pi$  becomes energetically allowed. The study of unstable particles is a difficult problem in a Euclidean time formalism [19]. We return to this topic later.

#### 2.2.4 *Experimental evidence for scalar mesons*

In full QCD, for the flavour-singlet states of any given  $J^{PC}$ , there will be mixing between the  $s\bar{s}$  state, the  $u\bar{u} + d\bar{d}$  state and the glueball as well as

with multi-meson channels. It may indeed turn out that no scalar meson in the physical spectrum is primarily a glueball — all states are mixtures of glue,  $q\bar{q}$ ,  $q\bar{q}q\bar{q}$ , etc.

To help with understanding the experimental situation [21], we first discuss the flavour non-singlet states, the  $a_0$  with isospin 1. The observed states are at 980 and 1450 MeV. The lighter state has dynamics which appears to be closely associated with the  $K\bar{K}$  threshold. The heavier state is not yet very well established but seems to be a candidate for a state mostly comprised of  $q\bar{q}$ , while the lighter state would be  $q\bar{q}q\bar{q}$ .

The flavour singlet states ( $f_0$ ) are more numerous. There is a very broad enhancement in the  $\pi\pi$  S-wave phase shift around 700 MeV (sometimes called the  $\sigma$ ), there is a state near the  $K\bar{K}$  threshold at 980 MeV and there are more states at 1370, 1500 and 1710 MeV. Again assuming that the state at 980 MeV is predominantly  $q\bar{q}q\bar{q}$ , this suggests that the three states in the 1300-1750 MeV energy range are admixtures of the glueball,  $u\bar{u} + d\bar{d}$  and  $s\bar{s}$ . The fact that there are indeed three states in this energy region close to the quenched glueball mass of 1600 MeV is the strongest evidence for the presence of a glueball. This has led to several phenomenological attempts [17, 22] to describe these three observed states in terms of the lattice input.

As we emphasised above, in full QCD on a lattice one just obtains values for the  $a_0$  and  $f_0$  masses. In the simplified case of  $N_f = 2$  flavours of degenerate quark, one does indeed find [8] two  $f_0$  states, and they can be interpreted as mixtures of the  $q\bar{q}$  and glueball states with the  $q\bar{q}$  state having the properties found for the  $a_0$ .

One useful lattice input would be a determination of the  $a_0$  mass as the quark mass is varied in full QCD, especially because of the problems of determining the  $a_0$  mass in the quenched approximation. At present, the full QCD studies [7, 8] are limited to relatively coarse lattice spacing, so the continuum limit is not close. Furthermore, as the quark mass is reduced the  $a_0$  can decay (to  $\pi\eta$ ) and this will influence the lattice analysis [23].

### 2.3 Hybrid Mesons

A hybrid meson is a meson in which the gluonic degrees of freedom are excited non-trivially. The most direct sign of this would be a spin-exotic meson, since that could not be created from a  $q\bar{q}$  state with unexcited

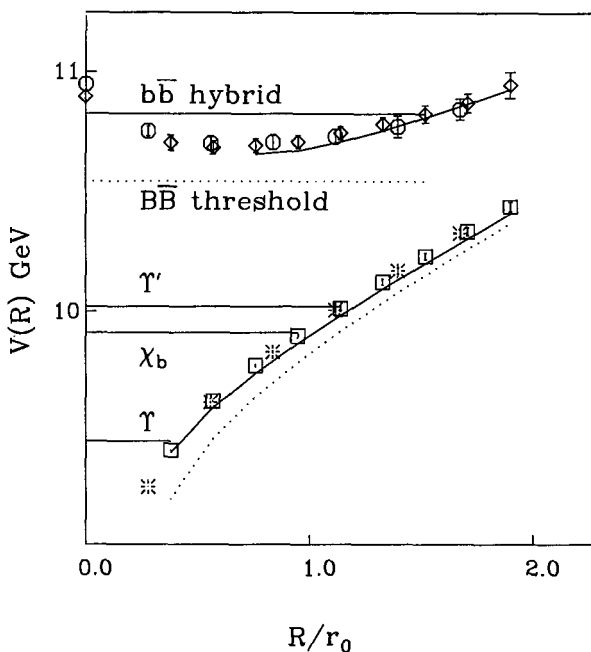


Fig. 2.3 The potential energy between static quarks at separation  $R$  (in units of  $r_0 \approx 0.5$  fm) [25]. The symmetric gluonic field configuration is shown by the lower points while the  $\Pi_u$  excited gluonic configuration is shown above. The energy levels in these potentials for  $b$  quarks are shown using the adiabatic approximation.

glue. A spin-exotic meson could, however, be a  $q\bar{q}q\bar{q}$  or meson-meson state and that possibility will be discussed. We first discuss hybrid mesons with static heavy quarks where the description can be thought of as an excited colour string. The situation concerning light quark hybrid mesons is then summarised

### 2.3.1 Heavy quark hybrid mesons

Consider  $Q\bar{Q}$  states with static quarks in which the gluonic contribution may be excited. We classify the gluonic fields according to the symmetries of the system. This discussion is very similar to the description of electron wave functions in diatomic molecules. The symmetries are (i) rotation around the separation axis  $z$  with representations labelled by  $J_z$  (ii)  $CP$  with representations labelled by  $g(+)$  and  $u(-)$  and (iii)  $CR$ . Here  $C$

interchanges  $Q$  and  $\bar{Q}$ ,  $P$  is parity and  $\mathcal{R}$  is a rotation of  $180^\circ$  about the mid-point around the  $y$  axis. The  $C\mathcal{R}$  operation is only relevant to classify states with  $J_z = 0$ . The convention is to label states of  $J_z = 0, 1, 2$  by  $\Sigma, \Pi, \Delta$  respectively. The ground state ( $\Sigma_g^+$ ) will have  $J_z = 0$  and  $CP = +$ .

The exploration of the energy levels of other representations has a long history in lattice studies [24, 25] — see Fig. 2.3. The first excited state is found to be the  $\Pi_u$ . This can be visualised as the symmetry of a string bowed out in the  $x$  direction minus the same deflection in the  $-x$  direction (plus another component of the two-dimensional representation with the transverse direction  $x$  replaced by  $y$ ), corresponding to flux states from a lattice operator which is the difference of  $\sqcup$ -shaped paths from quark to antiquark of the form  $\sqcap - \sqcup$ .

The picture of the gluon flux between the static quarks suggests that the excited states of this string may approximate the excited potentials found from the lattice. In the simplest string theory, the first excited level has  $\Pi_u$  symmetry and is at energy  $\pi/R$  above the ground state. This is indeed approximately valid and a closer approximation is to use a relativistic version [26], namely  $E_m(R) = [\sigma^2 R^2 + 2\pi\sigma(m - 1/12)]^{1/2}$  for the  $m$ -th level, with  $m = 0, 1, 2, \dots$  — see also Ref. [27] for a recent comparison of this expression.

Recent lattice studies [27] have used an asymmetric space/time spacing which enables excited states to be determined comprehensively. These results confirm the finding that the  $\Pi_u$  excitation is the lowest lying and hence of most relevance to spectroscopy.

From the potential corresponding to these excited gluonic states, one can determine the spectrum of hybrid quarkonia using the Schrödinger equation in the Born-Oppenheimer approximation. This approximation will be good if the heavy quarks move very little in the time it takes for the potential between them to become established. More quantitatively, we require that the potential energy of gluonic excitation is much larger than the typical energy of orbital or radial excitation. This is indeed the case [24], especially for  $b$  quarks. Another nice feature of this approach is that the self energy of the static sources cancels in the energy difference between this hybrid state and the  $Q\bar{Q}$  states. Thus the lattice approach gives directly the excitation energy of each gluonic excitation.

The  $\Pi_u$  symmetry state corresponds to excitations of the gluonic field in quarkonium called magnetic (with  $L^{PC} = 1^{+-}$ ) and pseudo-electric

Table 2.2 Heavy quark hybrid meson spin-parity combinations  $J^{PC}$  for singlets( $S = 0$ ) and triplets( $S = 1$ ), where  $J = L + S$ .

$L^{PC}$	$S = 0$	$S = 1$		
$1^{-+}$	$1^{--}$	$0^{-+}$	$1^{-+}$	$2^{-+}$
$1^{--}$	$1^{++}$	$0^{+-}$	$1^{+-}$	$2^{+-}$

(with  $1^{-+}$ ) in contrast to the usual P-wave orbital excitation which has  $L^{PC} = 1^{--}$ . Thus we expect different quantum number assignments from those of the gluonic ground state. Indeed combining with the heavy quark spins, we get the degenerate set of 8 states shown in Table 2.2. Note that of these,  $J^{PC} = 1^{-+}$ ,  $0^{+-}$  and  $2^{+-}$  are spin-exotic and hence will not mix with  $Q\bar{Q}$  states. They thus form a very attractive goal for experimental searches for hybrid mesons.

The eightfold degeneracy of the static approach will be broken by various corrections. As an example, one of the eight degenerate hybrid states is a pseudoscalar with the heavy quarks in a spin triplet. This has the same overall quantum numbers as the S-wave  $Q\bar{Q}$  state ( $\eta_b$ ) which, however, has the heavy quarks in a spin singlet. So any mixing between these states must be mediated by spin dependent interactions. These spin dependent interactions will be smaller for heavier quarks. It is of interest to establish the strength of these effects for  $b$  and  $c$  quarks. Another topic of interest is the splitting between the spin-exotic hybrids which will come from the different energies of the magnetic and pseudo-electric gluonic excitations.

One way to go beyond the static approach is to use the NRQCD approximation which then enables the spin dependent effects to be explored. One study [27] finds that the  $L^{PC} = 1^{+-}$  and  $1^{-+}$  excitations have no statistically significant splitting although the  $1^{+-}$  excitation does lie a little lighter. This would imply, after adding in heavy quark spin, that the  $J^{PC} = 1^{-+}$  hybrid was the lightest spin-exotic. Also a relatively large spin splitting was found [28] among the triplet states considering, however, only magnetic gluonic excitations. Another study [29] explores the mixing of non spin-exotic hybrids with regular quarkonium states via a spin-flip interaction using lattice NRQCD.

Confirmation of the ordering of the spin-exotic states also comes from

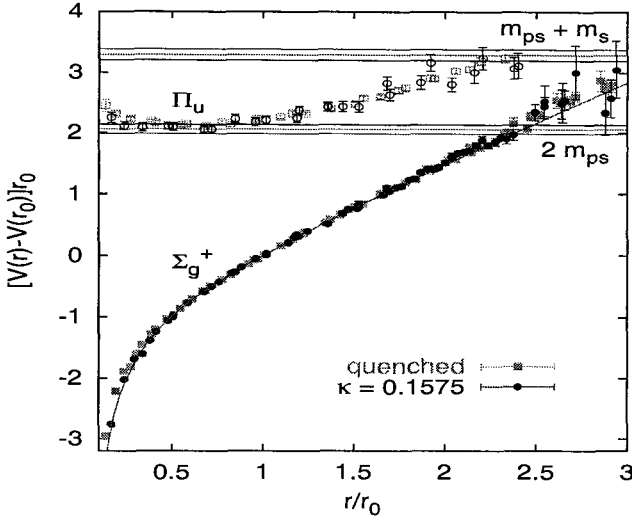


Fig. 2.4 The potential energy for quenched and 2 flavours of sea quark for the ground state and first excited gluonic state [6].

lattice studies with propagating quarks [30, 31, 32] which are able to measure masses for all 8 states. We discuss that evidence in more detail below.

Because of the similarity of the lightest hybrid wavefunction with that of the  $2S$  state (which is radially excited), it is convenient to quote mass differences between these states. Within the quenched approximation, the lattice evidence for  $b\bar{b}$  quarks points to a lightest hybrid spin-exotic with  $J^{PC} = 1^{-+}$  at an energy given by  $(m_H - m_{2S})r_0 = 1.8$  (static potential [25]); 1.9 (static potential [27], NRQCD [28]); 2.0 (NRQCD [27]). These results can be summarised as  $(m_H - m_{2S})r_0 = 1.9 \pm 0.1$ . Using the experimental mass of the  $\Upsilon(2S)$ , this implies that the lightest spin-exotic hybrid is at  $m_H = 10.73(7)$  GeV including a 10% scale error. Above this energy there will be many more hybrid states, many of which will be spin-exotic.

The results from a study with  $N_f = 2$  flavours of sea-quarks show very little change in the static potential (see Fig. 2.4) and also relatively little change in NRQCD determinations [28] of mass ratios such as  $(m_H - m_{2S})/(m_{1P} - m_{1S})$ . Expressed in terms of  $r_0$  (using  $r_0 = 1.18/\sqrt{\sigma}$ ) this gives  $(m_H - m_{2S})r_0 = 2.4(2)$ , however. This is significantly larger than the quenched result and, using the  $1P - 1S$  mass difference to set the scale, yields a prediction [28] for the lightest hybrid mass of 11.02(18) GeV.

### 2.3.2 Hybrid meson decays

Within this static quark framework, one can explore the decay mechanisms. One special feature is that the symmetries of the quark and colour fields about the static quarks must be preserved exactly in decay. This has the consequence that the decay from a  $\Pi_u$  hybrid state to the open- $b$  mesons ( $B\bar{B}$ ,  $B^*\bar{B}$ ,  $B\bar{B}^*$ ,  $B^*\bar{B}^*$ ) will be forbidden [41, 33] if the light quarks in the  $B$  and  $B^*$  mesons are in an  $S$ -wave relative to the heavy quark (since the final state will have the light quarks in either a triplet with the wrong  $CP$  or a singlet with the wrong  $J_z$  where  $z$  is the interquark axis). The decay to  $B^{**}$ -mesons with light quarks in a P-wave is allowed by symmetry but not energetically.

The only allowed decays are when the hybrid state de-excites to a non-hybrid state with the emission of a light quark-antiquark pair. Since the  $\Pi_u$  hybrid state has the heavy quark-antiquark in a triplet P-wave state, the resulting non-hybrid state must also be in a triplet P-wave since the heavy quarks do not change their state in the limit of very heavy quarks. Thus the decay for  $b$  quarks will be to  $\chi_b + M$  where  $M$  is a light quark-antiquark meson in a flavour singlet. This proceeds by a disconnected light quark diagram and it would be expected [34] that the scalar or pseudoscalar meson channels are the most important (*i.e.* they have the largest relative OZI-rule violating contributions). Lattice estimates [33] of these transitions have been made and the dominant mode (with a width of around 100 MeV) is found to be with  $M$  as a scalar meson, namely  $H \rightarrow \chi_b + f_0$ .

These estimates are in the static quark limit, in which the spin-exotic and non-spin-exotic hybrid mesons are degenerate. For the latter, however, the interpretation of any observed states is less clear cut, since they could be conventional quark-antiquark states. Moreover, the non-spin-exotic hybrid mesons can mix directly (*i.e.* without emission of any meson  $M$ ) with conventional quark-antiquark states once one takes into account corrections (of order  $1/M_Q$ ) to the static approximation applicable for heavy quarks with physical masses.

It is encouraging that the decay width comes out as relatively small, so that the spin-exotic hybrid states should show up experimentally as sufficiently narrow resonances to be detectable. This decay analysis does not take into account heavy quark motion or spin-flip and these effects will be significantly more important for charm quarks than for  $b$ -quarks.

### 2.3.3 Light quark hybrid mesons

I now focus on lattice results for hybrid mesons made from light quarks using fully relativistic propagating quarks. There will be no mixing with  $q\bar{q}$  mesons for spin-exotic hybrid mesons and these are of special interest. The first study of this area was by the UKQCD Collaboration [30] who used operators motivated by the heavy quark studies referred to above to study all 8  $J^{PC}$  values coming from  $L^{PC} = 1^{+-}$  and  $1^{-+}$  excitations. The resulting mass spectrum gives the  $J^{PC} = 1^{-+}$  state as the lightest spin-exotic state. Taking account of the systematic scale errors in the lattice determination, a mass of 2000(200) MeV is quoted for this hybrid meson with  $s\bar{s}$  light quarks. Although not directly measured, the corresponding light quark hybrid meson would be expected to be around 120 MeV lighter.

A second lattice group has also evaluated hybrid meson spectra with propagating quarks from quenched lattices. They obtain masses of the  $1^{-+}$  state with statistical and various systematic errors of 1970(90)(300) MeV, 2170(80)(100)(100) MeV and 4390(80)(200) MeV for  $n\bar{n}$ ,  $s\bar{s}$  and  $c\bar{c}$  quarks respectively [31]. For the  $0^{+-}$  spin-exotic state they have a noisier signal but evidence that it is heavier. They also explore mixing matrix elements between spin-exotic hybrid states and 4 quark operators.

The first analysis [32] to determine the hybrid meson spectrum using full QCD used Wilson quarks. The sea quarks used had several different masses and an extrapolation was made to the limit of physical sea quark masses, yielding a mass of 1.9(2) GeV for the lightest spin-exotic hybrid meson, which again was found to be the  $1^{-+}$ . In principle this calculation should take account of sea quark effects such as the mixing between such a hybrid meson and  $q\bar{q}q\bar{q}$  states such as  $\eta\pi$ , although it is possible that the sea quark masses used are not light enough to explore these features.

A recent dynamical quark study from 2+1 flavours of improved staggered quarks has also produced results [36]. They also compare their results with quenched calculations and find no significant difference, except that the ambiguity in fixing the lattice energy scale is better controlled in the dynamical simulation since different reference observables are closer to experiment. Their summary result for the  $1^{-+}$  hybrid with strange quarks is  $2100 \pm 120$  MeV, in agreement with earlier results. They note that the energies of two-meson states [such as  $\pi + b_1$  or  $K + K(1^+)$ ] with the hybrid meson quantum numbers are close to the energies they obtain. This suggests that these two-particle states, which are allowed to mix in a dynamical

quark treatment, may be influencing the masses determined. A study of hybrid meson transitions to two particle states is needed to illuminate this area, using techniques such as those used for heavy quark hybrid decay [33] and decays of light quark vector mesons [35].

The lattice calculations [30, 31, 32, 36, 37] of the light hybrid spectrum are in good agreement with each other. They imply that the natural energy range for spin-exotic hybrid mesons is around 1.9 GeV. The  $J^{PC} = 1^{-+}$  state is found to be lightest. It is not easy to reconcile these lattice results with experimental indications [38] for resonances at 1.4 GeV and 1.6 GeV, especially the lower mass value. Mixing with  $q\bar{q}q\bar{q}$  states such as  $\eta\pi$  is not included for realistic quark masses in the lattice calculations. Such effects of pion loops (both real and virtual) have been estimated in chiral perturbation theory based models [39] and they could potentially reconcile some of the discrepancy between lattice mass estimates (with light quarks which are too heavy) and those from experiment. This can be interpreted, dependent on one's viewpoint, as either that the lattice calculations are incomplete or as an indication that the experimental states may have an important meson-meson component in them.

The light quark technique of using relativistic propagating quarks can also be extended to charm quarks, as was note above [31]. Another group has explored the charm quark hybrid states also using a fully relativistic action, albeit with an anisotropic lattice formulation [40]. Their quenched study is in agreement with the isotropic lattice result quoted above, finding a mass value of 4.428(41) GeV in the continuum limit for the  $1^{-+}$  hybrid where the scale is set by the  $^1P_1 - 1S$  mass splitting (458.2 MeV experimentally) in charmonium. Their result is also consistent with that from NRQCD methods [28] applied to this case. These results all have the usual caveat that in quenched evaluations the overall mass scale of the energy difference from the  $1S$  state at 3.067 GeV is uncertain to 10% or so (for example the  $(2S - 1S)/(^1P_1 - 1S)$  is found to be 15% higher than experiment) which is a major source of systematic error (approximately  $\pm 140$  MeV). They also produce estimates for other charmonium spin-exotic states:  $0^{+-}$  at 4.70(17) GeV and  $2^{+-}$  at 4.89(9) GeV. The  $0^{--}$  state is not resolved.

Thus masses near 4.4 GeV are found for the charmonium  $1^{-+}$  state using relativistic quarks. The non-relativistic approach using NRQCD is expected to have big systematic errors for quarks as light as charm, but results [28] do agree with this value. The heavy quark effective theory approach has a leading term which corresponds to a static heavy quark, resulting in an

estimate [25] of the spin-exotic charm state mass of 4.0 GeV. Here again the systematic error is potentially large for charm quarks.

## 2.4 Hadronic Molecules

By exotic state we mean any state which is not dominantly a  $q\bar{q}$  or  $qqq$  state. For example, a state made from hadrons bound in a molecule would be exotic. Examples of hadronic molecules have been known for a long time: the deuteron is a proton-neutron molecule for example. It is very weakly bound (2 MeV) and is quite extended. It is more efficiently described in terms to a neutron and a proton than as six quarks.

The residual hadronic interaction, the force between two colour-singlet hadrons, is much weaker than the colour force between quarks. Although it is called a ‘strong interaction’, it is relatively weak. At large distance, it will be dominated by the exchange of the lightest hadrons allowed (typically one or two pions). For example, the scale of nuclear binding is around 8 MeV whereas the gluonic forces binding three quarks to make a nucleon contribute most of its mass of 938 MeV. For this reason lattice methods need to be developed specially to tackle this problem. Basically, one is interested in binding energies, so it is the energy difference between the two hadrons and the hadronic molecule that is of interest. This difference can sometimes be determined better than the total energy itself. Even so, the detailed dynamics of such molecular states will depend on the long range forces (typically one or two pion exchange) and this will be modified considerably in lattice studies with light quark masses which are too heavy (typically down to 50% of the strange quark mass only). So only qualitative input can be obtained from the lattice, but this can still be used to validate models.

Because of the small binding energy and the dominance of pion exchange in the binding, it is not feasible to obtain the deuteron binding direct from QCD using lattice methods at present. There has been speculation that other di-baryon systems might be more strongly bound: the H dibaryon (a  $\Lambda\Lambda$  state) being the best known. If it were strongly bound then it might be stable to weak decay which might have astrophysical consequences. The current status of lattice studies [42] is that finite box size effects are large but there is no convincing lattice evidence that this state is bound. At the largest volumes studied, with  $L \approx 4$  fm, in quenched simulations it is found

that the ratio  $(m_H/2m_\Lambda) - 1$  is positive and in the range 5 to 15 %.

Other molecular states involving two hadrons have been conjectured. Several meson resonances are known which are closely connected with nearby thresholds: the  $\Lambda(1405)$  which is just below the  $\bar{K}N$  threshold and the  $a_0(980)$  and  $f_0(980)$  which are close to the  $K\bar{K}$  threshold. Another state close to a threshold is the  $N(1535)$  which is just above the  $\eta N$  threshold. Again lattice studies are not able to shed very much light directly on these states since the quark masses used in the lattice studies are unphysical. However, if they are not produced in a lattice study which explores  $q\bar{q}$  and  $qqq$  states, this may help to support the conclusion that they are primarily molecular in structure.

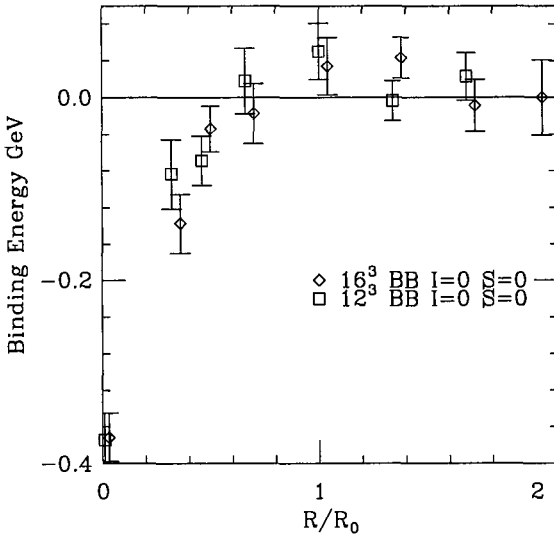


Fig. 2.5 The binding energy [43] between two heavy-light mesons (with static heavy quarks and light quarks of mass corresponding to strange) at separation  $R$  (in units of  $r_0 \approx 0.5$  fm) with the two light quarks having  $I = 0$  and  $S = 0$ .

One case which is relatively easy to study is the  $BB$  system, idealised as two static quarks and two light quarks. Then a potential as a function of the separation  $R$  between the static quarks can be determined. Because the static quark spin is irrelevant, the states can be classified by the light quark spin and isospin. Lattice results [43] (using a light quark mass close to strange) have been obtained for the potential energy for  $I_q = 0, 1$  and

$S_q = 0, 1$ . For very heavy quarks, a potential below  $2M_B$  will imply binding of the  $BB$  molecules with these quantum numbers and  $L = 0$ . For the physically relevant case of  $b$  quarks of around 5 GeV, the kinetic energy will not be negligible and the binding energy of the  $BB$  molecular states is less clear cut. One way to estimate the kinetic energy for the  $BB$  case with reduced mass circa 2.5 GeV is to use analytic approximations to the potentials found. For example the  $I_q, S_q = (0, 0)$  case (see Fig. 2.5) shows a deep binding at  $R = 0$  which can be approximated as a Coulomb potential of  $-0.1/R$  in GeV units. This will give a di-meson binding energy of only 10 MeV. For the other interesting case shown in Fig. 2.6,  $(I_q, S_q) = (0, 1)$ , a harmonic oscillator potential in the radial coordinate of form  $-0.04[1 - (r - 3)^2/4]$  in GeV units leads to a kinetic energy which completely cancels the potential energy minimum, leaving zero binding. This harmonic oscillator approximation lies above the estimate of the potential, so again we expect weak binding of the di-meson system.

Because of these very small values for the di-meson binding energies, one needs to retain corrections to the heavy quark approximation to make more definite predictions, since these corrections are known to be of magnitude 46 MeV from the  $B, B^*$  splitting. It will also be necessary to extrapolate

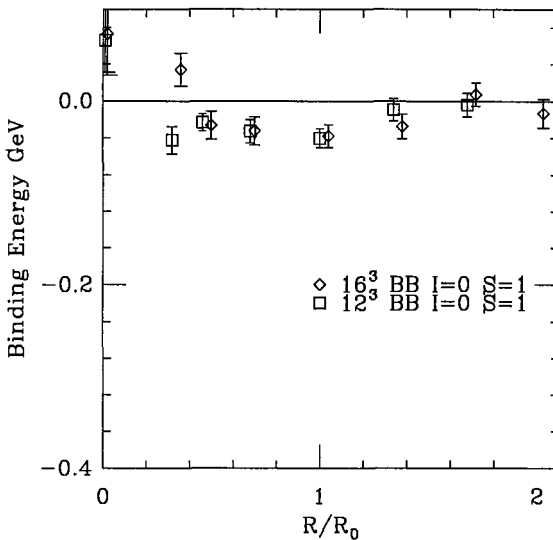


Fig. 2.6 Same as Fig. 2.5 but for  $I = 0$  and  $S = 1$ .

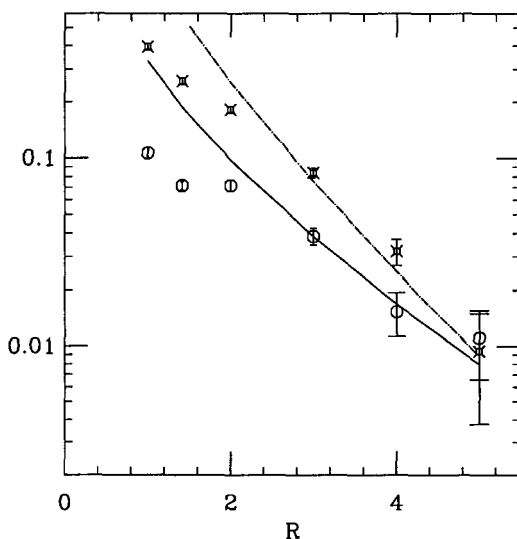


Fig. 2.7 The contribution [43] to the binding energy for the spin and isospin combinations corresponding to  $\pi$  exchange (octagons) and  $\rho$  exchange (fancy squares). The solid line gives the  $\pi$  exchange contribution which is normalised by the  $B^*B\pi$  coupling. The  $\rho$  exchange prediction has a free normalisation and is shown by the dash-dotted line. The results are plotted versus interquark separation  $R$  (in units of  $a \approx 0.17$  fm).

the light quark mass from strange to the lighter  $u$ ,  $d$  values to make more definite predictions about the binding of  $BB$  molecules.

Models for the binding of two  $B$  mesons involve, as in the case of the deuteron, pion exchange. The lattice study [43] is able to make a quantitative comparison of lattice pion exchange with the data described above using lattice determinations of the  $B^*B\pi$  coupling [44] and excellent agreement is obtained at larger  $R$  values as shown in Fig. 2.7, as expected.

## 2.5 Conclusions and Outlook

Quenched lattice QCD is well understood and accurate predictions in the continuum limit are increasingly becoming available. The lightest glueball is scalar with mass  $m(0^{++}) = 1611(30)(160)$  MeV where the second error is an overall scale error. The excited glueball spectrum is known too. The quenched approximation also gives information on quark-antiquark scalar mesons and their mixing with glueballs. This determination of the mixing

in the quenched approximation also sheds light on results for the spectrum directly in full QCD where the mixing will be enabled. In full QCD, the scalar meson masses are determined directly but there is no concept of a glueball as such, much as in the experimental case. Additional work is needed to reduce the lattice spacing, or use improved actions, to explore the continuum limit for scalar mesons in full QCD. There is also some lattice information on the hadronic decay amplitudes of glueballs and this is an area where further study may be anticipated.

For hybrid mesons, there will be no mixing with  $q\bar{q}$  for spin-exotic states and these are the most useful predictions. The  $J^{PC} = 1^{-+}$  state is expected in the range 10.7 to 11.0 GeV for  $b$  quarks, 2.0(2) GeV for  $s$  quarks and 1.9(2) GeV for  $u, d$  quarks. Mixing of spin-exotic hybrids with  $q\bar{q}q\bar{q}$  or equivalently with meson-meson is allowed and will modify the predictions from the quenched approximation. A first lattice study has been made of hybrid meson decays. For heavy quarks, the dominant mode is string de-excitation to  $\chi + f_0$  where  $f_0$  is a flavour singlet scalar meson (or possibly two pions in this state). The magnitude of the decay rate is found to be of order 100 MeV, so this decay mode should still leave a detectably narrow resonance to be observed.

The topic of possible multi-quark bound states is difficult because the scale of the expected binding energies is a few MeV and this small value is a challenge for lattice studies. As an example, some evidence was presented for a possible  $B_s B_s$  molecular state.

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