Preface

The aim of this volume is to provide an introduction to Lattice Quantum Chromodynamics (LQCD) for readers that are non-specialists in the field. In particular, we have in mind advanced undergraduates and graduate students — both theorists and experimentalists — who have a background in nuclear and/or particle physics. This leads to a very limited selection being made from the many topics that are covered by LQCD. The particular topics chosen have, in most cases, analogies in more conventional nuclear and particle physics — good examples being the chapters on the interquark potential and the interactions between hadrons, both of which can be compared with the nucleon-nucleon potential and the interactions between nucleon clusters.

Why should a nuclear/particle physicist ever be interested in LQCD? This is certainly a valid question for someone with little or no knowledge of the field. For why should such a person spend their time reading about some subject that could be dead within a few years? However, in the case of LQCD, it is firmly believed that this is indeed a subject which is here to stay and, also, is expected to be continuously developed for many years to come. The reason for being so confident about this prediction can be summarised by the following phrase — "QCD is thought to be the theory of strong interactions" — words that have appeared in the literature many times over the last 30 years. Unfortunately, to prove this in its full generality has yet to be achieved. In practice, the theory can only be checked either in limiting cases — such as perturbation theory — or by some purely numerical approach — such as LQCD.

The perturbation theory approach exploits the fact that the basic QCD interaction exhibits asymptotic freedom. This states that the strength (α) of the fundamental interaction between quarks and gluons becomes *weaker* as the momentum involved becomes *larger*. Therefore, in principle, high energy processes can be described by evaluating contributions to sufficiently high order in (α). This procedure has been applied to many cases with great success and is one of the main reasons for the above optimistic phrase about QCD being the theory of strong interactions. Unfortunately, in most situations the above perturbation limit is not applicable, since the coupling (α) is of the order unity.

In these non-perturbative regimes, attempts to deal directly with the basic QCD Lagrangian [$\mathcal{L}$(QCD)] lead to singular integrals. There are ways to regulate these using, for example, cut-offs. But the outcome is an infinite number of such integrals rendering the problem as essentially still intractable. This was overcome by Wilson in 1974 through replacing $\mathcal{L}$(QCD) by $\mathcal{L}$(Lattice) — a discretized form on a 4-dimensional space-time lattice. The effect of the lattice is essentially to introduce, in a systematic manner, a high momentum cut-off of π/a, where a is the lattice spacing. This now results in a finite number of well defined integrals — but they need to be evaluated numerically. In $\mathcal{L}$(Lattice) the quark fields only exist on the lattice sites, whereas the gluon field only exists on the links between these sites. Of course, to extract observable physical quantities, the limit $a \to 0$ must eventually be taken. However, it should be emphasized that these final results are *exact* solutions of QCD — a point that was well summarised by Martin Lüscher (hep-ph/0211220) with the words: "In general numerical simulations have the reputation of being an approximate method that mainly serves to obtain qualitative information on the behaviour of complex systems. This is, however, not so in lattice QCD, where the simulations produce results that are exact (on a given lattice) up to statistical errors. The systematic uncertainties related to the non-zero lattice spacing and the finite lattice volume then still need to be investigated, but these effects are theoretically well understood and can usually be brought under control". In most cases the size of these errors and uncertainties are indeed being continuously reduced by the access to computer resources that are ever increasing in speed and memory.

In the above, I say that there is an analogy between nuclear physics and LQCD, since both use the concept of potentials. However, it should be said immediately that this analogy can not be carried too far. For example, the nucleon-nucleon potential $V(NN)$ plays a much more central rôle in the development of nuclear physics than the interquark potential $V(QQ)$ does in QCD. This is clearly seen by opening books on nuclear physics, where often one of the first chapters is a detailed discussion of $V(NN)$ followed by its use in, for example, the Faddeev equations for an exact description of the three nucleon systems He^3 and H^3. Also nuclei, upto $A \approx 16$, can be understood using variational techniques. In all cases the models are expressed *directly* in terms of $V(NN)$ with all its complexities — such as tensor components and non-localities — and the results are essentially *exact.* This does not mean to say that these results agree with experiment, since there can be corrections due to three-body forces, relativistic effects, *etc.* For heavier nuclei with $A > 16$ (including nuclear and neutron matter) $V(NN)$ is first converted into an effective interaction $V_{\mathrm{Eff}}(NN)$ by means of Brueckner-like techniques. These treat the interaction between two specific nucleons exactly but with the effects of the other nucleons only being included in an approximate way. Models for these heavier nuclei are then expressed in terms of $V_{\mathrm{Eff}}(NN)$. The purpose of this digression is to emphasize that a major goal of nuclear physics is to explain the structure of all nuclei in terms of the basic interaction $V(NN)$. In contrast, this program is *not* generally considered to be a goal in QCD. There $V(QQ)$ is, indeed, a topic of active research, but it is not usually considered as a stepping stone in the understanding of multi-quark systems. Even for three-quark systems — the baryons — the starting point is the basic latticized lagrangian $\mathcal{L}$(Lattice) mentioned earlier. This approach treats the quarks alongwith *explicit* gluons — unlike the use of $V(NN)$ where the mesons only appear implicitly. Having just played down the rôle of $V(QQ)$ for the understanding of multi-quark systems, I should add that attempts have been made to bridge the gap between few- and multi-quark systems by way of interquark potentials. But, at present, these are still in their infancy with most of such models simply mimicking successful nuclear physics approaches.

This volume is made up from five separate chapters each of which has its counterpart in nuclear/particle physics. Also each chapter is essentially self-contained so that there is unavoidably a certain amount of repetition of, in particular, the basics of LQCD. However, this I consider to be a positive feature, since each author emphasizes different aspects that are more relevant for their particular topic.

- Chapter 1 discusses the spectroscopy of mesons and baryons — the 2 and 3 quark analogies to the deuteron and the He^3, H^3 nuclei. Unlike nuclear/particle physics, these few-quark systems are treated directly and not in terms of $V(QQ)$. Also they exhibit a much richer spectrum, since they can be constructed from combinations of 5 different types of quark, the u, d, s, c and b quarks, whereas nuclei in general are constructed from only protons and neutrons. An exception to the latter are a few hypernuclei involving strange baryons. In contrast, there are seen charmed and bottom baryons, such as the $\Omega_c^0 = ssc$ and $\Xi_b^- = dsb$. Furthermore, these are expected to exist not only in S-wave states but also in higher partial waves — again a difference to the few-baryon systems.
- Chapter 2 discusses exotic few-quark states. In Chapter 1, the states are characterised as different combinations of the 5 types of quark with no reference to the gluon field that generates the interaction between the quarks — it being assumed to be in its ground state. In this chapter the states are, in addition, characterised by the excitation of the gluon field. This explicit manifestation of the gluon field is unavoidable in QCD, but has so far not been convincingly seen experimentally. Again this is very different to the nuclear/particle physics situation where the meson fields generating $V(NN)$, for example the π and the ρ plus their many excitations, *e.g.* the $\pi(1300)$ and the $\rho(1450)$, are well documented. In this chapter, since the term "exotic" is taken to mean states not included in the naïve quark model involving simply 2 or 3 quarks, the subject of the molecular states of two mesons is also reviewed.
- Chapter 3 discusses the basic interquark potential $V(QQ)$. This is in analogy with many text books in nuclear physics that start with the nucleon-nucleon potential $V(NN)$. Also the terminology involving spin dependence and radial forms is quite similar. However, the analogy essentially ends at that point, since — unlike the nuclear/particle physics counterpart — the study of multiquark systems generally does not build directly upon this knowledge of $V(QQ)$.
- Chapter 4 discusses the interactions between few-quark systems — the analogies to deuteron-deuteron or He^3-He^3 scattering. Since this involves so many quarks, direct LQCD calculations, with a realistic vacuum containing quark-antiquark pairs, can not yet be

carried out using sufficiently light quarks and so, as the authors themselves say, the field is still "in an exploratory phase". Even so it is in the direction that is necessary for bridging the gap between QCD and nuclear/particle physics.

- Chapter 5 has a different goal compared with the earlier chapters. Here the main aim is to generate lattice data that can help in the construction of models and not necessarily for direct comparison with experiment. In a sense, it is a continuation of Chapter 4 by discussing more generally the quest for bridges between QCD and nuclear/particle physics. This requires lattice calculations with multiquark systems — mainly with 4 quarks. There is no question that models are needed for understanding multiquark systems, where the number of quarks is much larger than four, since LQCD calculations are not feasible for general multiquark systems. In fact, there are on–the–market many multiquark models that are "inspired" by QCD. Unfortunately, few are actually "based" on QCD — the inspirations mainly coming from successful multinucleon models.

Naturally, as with most projects involving authors distributed world-wide, there have been some "communication" problems. Fortunately, these were eventually resolved but — in one case — not without considerable cajoling. Several of the contributors actually expressed their pleasure for the opportunity to review their work for non-experts. But, of course, hopefully the volume will prove most useful for the audience it is primarily intended — the non-experts.

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