Preface

Motivation

This book deals with theories and algorithms for drawing planar graphs. Graph drawing has appeared as a lively area in computer science due to its applications in almost all branches of science and technology. Many researchers have concentrated their attention on drawing planar graphs for the following reasons:

- drawings of planar graphs have no edge crossings, and look nice;
- drawings of planar graphs have practical applications in VLSI floorplanning and routing, architectural floorplanning, displaying RNA structures in bioinformatics, etc.; and
- algorithms for drawing planar graphs can be successfully used for drawing a nonplanar graph by transforming it into a similar planar graph.

During the last two decades numerous results have been published on drawing planar graphs. For example, in 1990 it was shown that every planar graph of n vertices has a straight-line drawing on a grid of area $O(n^2)$. This result solved the open question for about four decades whether a planar graph has a straight line drawing on a grid of a polynomial area. Many algorithms have been developed to produce drawings of planar graphs with different styles to fulfill different application needs. While developing these algorithms, many elegant theories on the properties of planar graphs have been discovered, which have applications in solving problems on planar graphs other than graph drawing problems. For example, Schnyder introduced a "realizer" to produce straight line drawings of planar graphs, but later a realizer is used to solve the "independent spanning tree problem" of a certain class of planar graphs. A "canonical ordering" which was introduced by de Fraysseix *et al.* is later used to solve a "graph partitioning problem." On the other hand, many established graph theoretic results have been successfully used to solve graph drawing problems. For example, the problem of orthogonal drawings of plane graphs with the minimum number of bends is solved by reducing the problem to a network flow problem.

Recently, it appeared to us that a systematic and organized book containing these many results on planar graph drawings can help students and researchers of computer science to apply the results in appropriate areas. For example, we observed that people working with VLSI floorplanning by rectangular dual did not notice Thomassen's result on rectangular drawings of plane graphs. In our opinion the theory and algorithms are complementary to each other in the research of planar graph drawings. We have thus tried to include in the book most of the important theorems and algorithms that are currently known for planar graph drawing. Furthermore, we have tried to provide constructive proofs for theorems, from which algorithms immediately follow.

Organization of the Book

This book is organized as follows.

Chapter 1 is the introduction of graph drawing. It introduces different drawing styles of planar graphs, and presents properties of graph drawing and some applications of graph drawing.

Chapter 2 deals with graph theoretic fundamentals.

Chapter 3 provides algorithmic fundamentals.

Chapter 4 describes straight line drawings of planar graphs on an integer grid. We present both the famous results of de Fraysseix *et al.* and Schnyder on straight-line drawings of planar graphs in this chapter.

Chapter 5 focuses on convex drawings of planar graphs. In this chapter we present the results of Tutte, Thomassen and Chiba *et al.* on characterization of planar graphs with convex drawings. We also include recent results on convex grid drawings by Kant and Chrobak, and Miura *et al.*

Chapter 6 deals with rectangular drawings of planar graphs. In this chapter we present a technique of Miura *et al.* for reducing a rectangular drawing problem to a matching problem. We present Thomassen's result on rectangular drawings of plane graphs, and describe a generalization of

Thomassen's result given by Rahman *et al.* We also present a necessary and sufficient condition for a planar graph to have a rectangular drawing. Several algorithms for rectangular drawings are included in this chapter.

Chapter 7 deals with box-rectangular drawings of plane graphs. In this chapter we present a necessary and sufficient condition for a plane graph to have a box-rectangular drawing, and present a linear algorithm for finding a box-rectangular drawing of a plane graph.

Chapter 8 discusses orthogonal drawings of plane graphs. In this chapter we present the results of Tamassia for solving the problem of finding a bend-minimum orthogonal drawing of a plane graph by reducing the problem to a network flow problem. We explain a linear algorithm for finding a bend-minimum orthogonal drawing of a triconnected cubic plane graph. In this chapter we also include a necessary and sufficient condition for a plane graph to have a no-bend orthogonal drawing.

Chapter 9 deals with octagonal drawings of plane graphs with prescribed face areas. In this chapter we show that every "good slicing graph" has an octagonal drawing where each face is drawn as a rectilinear polygon of at most eight corners and the area of each inner face is equal to a prescribed value. We also present a linear algorithm for finding such a drawing.

Appendix A presents planarity testing and embedding algorithms.

Use of the book

This book is suitable for use in advanced undergraduate and graduate level courses on Algorithms, Graph Theory, Graph Drawing, Information Visualization, and Computational Geometry. This book will serve as a good reference book for the researchers in the field of graph drawing. In this book many fundamental graph drawing algorithms are described with illustrations, which are helpful for software developers, particularly in the area of information visualization, VLSI design and CAD.

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