

Preface

Several physical phenomena are governed by partial differential equations. Obviously, this is not a mere chance. Indeed, scientists and technicians, following the scientific method introduced by Galilei, know very well that every natural event can be analyzed by means of a related mathematical model. This is very often based on partial differential equations. A celebrated and historical example is given by the Maxwell equations, which admirably describe the electro-magnetic field. Other emblematic, not less important, models are: Fourier (heat) equation, D'Alembert (wave) equation, Laplace/Poisson (potential) equation, Euler/Stokes (fluid) equations.

Mathematical modeling has been successively adopted by researchers working in several other fields and so, at present, partial differential equations can help to explain phenomena occurring in Biology, Medicine, Economy, Sociology, and so on.

During the last two hundred years, applied mathematician involved in the above topic have never ended. The first pioneers concentrated their efforts on the exact solutions. Next, the attention was focused on the qualitative analysis in order to obtain information on the solutions even when these could not be explicitly found. In the last sixty years, also the quantitative analysis has been strongly developed in parallel with the great growth of computers power. At present, numerical methods provide a powerful approach for solving partial differential equations and their knowledge becomes more and more a necessary cultural luggage of applied researchers and technicians.

This book is an elementary introduction to computational methods, based on the finite differences, for parabolic, hyperbolic and elliptic partial differential equations. The numerical discussion of each type of equation is always preceded by the introduction of models in Mechanics, which are

carefully derived. In particular, the role of the initial and boundary conditions is pointed out with reference to physical situations. The numerical analysis is concerned with the most used finite-difference methods for each type of equation. Several examples are solved and exercises proposed to give the Reader the opportunity to practice. Since there is no numerical method without approximations, the accuracy of the schemes is also analyzed. Special attention is devoted to the propagation of rounding errors, which inevitable arise during the computational process. This leads to discuss the basic concepts of stability, consistency and convergence, which are illustrated by inductive procedures and then formalized.

Dealing with applied numerical methods sooner or later requires that own programs have to be developed. Therefore, some examples are provided by using the C++ language. The classic C language was evolved after 1970 by B. W. Kernighan and D. M. Ritchie. C++, developed by B. Stroustrup in 1980s, is an extension of C. It provides the capabilities of object-oriented programming. It is considered the most powerful and flexible language. The Visual C++¹ compiler is quickly introduced and Windows² programs are built step by step. These programs can be easily fitted to other situations, with small changes. The codes of the programs developed in this book are also supplied with the enclosed CD-Rom.

Chapter 1 deals with the most used finite-difference approximations of derivatives. In particular, the forward, central and backward approximations are derived. Chapter 2 presents the classical model of heat conduction in solids, based on Fourier law. Also, phase-change problems are illustrated. Chapter 3 introduces the classical explicit method for the one-dimensional heat equation. The concepts of stability, consistency and convergence are applied. Developing programs in C++, related to the heat equation, is the topic of Chapters 4 and 5. Chapter 6 describes numerical methods for parabolic equations. Some nonlinear case is also considered and a melting problem is discussed. Furthermore, the classical explicit method is generalized to the heat equation in two and three space dimensions. A program related to this case is developed in Chapter 7. Chapter 8 is devoted to wave motions. At first, the wave equation is derived from the model of one-dimensional continuum. Next, we introduce the equations governing the motions of general continuous systems. As special cases the elastic and fluid media are examined in order to point out wave phenomena. Finally, a free-boundary value problem is presented. Explicit and implicit finite-

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difference methods for the wave equation are illustrated in Chapter 9. An example of third-order equation is provided. Chapter 10 deals with finite-difference methods for linear and nonlinear hyperbolic systems. Elliptic equations are considered in Chapter 11. The motions of fluids through porous media are examined. In Appendix A the classification of partial differential equations is given for second-order equations and general systems of first-order equations. Elements of Linear Algebra are briefly provided in Appendix B.

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