

CHAPTER 1

INTRODUCTION

It is generally accepted that quantum field theory is the appropriate framework for describing the strong, electromagnetic and weak interactions between elementary particles. As for the electromagnetic interactions, it has been known for a long time that they are described by a quantum gauge field theory. But that the principle of gauge invariance also plays a fundamental role in the construction of a theory for the strong and weak interactions has been recognized only much later. The unification of the weak and electromagnetic interactions by Glashow, Salam and Weinberg was a major breakthrough in our understanding of elementary particle physics. For the first time one had been able to construct a renormalizable quantum field theory describing simultaneously the weak and electromagnetic interactions of hadrons and leptons. The "electro-weak" theory of Glashow, Salam and Weinberg is based on a non-abelian $SU(2) \times U(1)$ gauge symmetry, which is broken down spontaneously to the $U(1)$ symmetry of the electromagnetic interactions. This breaking manifests itself in the fact that, in contrast to the massless photon, the particles mediating the weak interactions, i.e., the W^+ , W^- and Z^0 vector bosons, become massive. In fact they are very massive, which reflects the fact that the weak interactions are very short ranged. The detection of these particles constituted one of the most beautiful tests of the Glashow-Salam-Weinberg theory.

The fundamental fermions to which the vector bosons couple are the quarks and leptons. The quarks, which are the fundamental building blocks of hadronic matter, come in different "flavours". There are the "up", "down", "strange", "charmed", "bottom" and "top" quarks. The weak interactions can induce transitions between different quark flavours. For example, a "u" quark can convert into a "d" quark by the emission of a virtual W^+ boson. The existence of the quarks has been confirmed (indirectly) by experiment. None of them have been detected as free particles. They are permanently confined within the hadrons which are built from the different flavoured quarks and antiquarks. The forces which are responsible for the confinement of the quarks are the strong interactions. Theoretical considerations have shown, that the "up", "down", etc., quarks should come in three "colours". The strong interactions are flavour blind, but sensitive to colour. For this reason one calls the theory of strong interactions *Quantum Chromodynamics*, or in short, QCD. It is a gauge theory based on the unbroken non-abelian $SU(3)$ -colour group (Fritzsch and Gell-Mann, 1972; Fritzsch,

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Gell-Mann and Leutwyler, 1973). The number "3" reflects the number of colours carried by the quarks. Since there are eight generators of $SU(3)$, there are eight massless "gluons" carrying a colour charge which mediate the strong interactions between the fundamental constituents of matter. By the emission or absorption of a gluon, a quark can change its colour.

QCD is an asymptotically free theory (t'Hooft, 1972; Politzer, 1973; Gross and Wilczek, 1973). Asymptotic freedom tells us that the forces between quarks become weak for small quark separations. Because of this asymptotic freedom property it was possible for the first time to carry out quantitative perturbative calculations of observables in strong interaction physics which are sensitive to the short distance structure of QCD.* In particular it allowed one to study the Bjorken scaling violations observed in deep inelastic lepton nucleon scattering at SLAC. QCD is the only theory we know that can account for these scaling violations.

The asymptotic freedom property of QCD is intimately connected with the fact that it is based on a non-abelian gauge group. As a consequence of this non-abelian structure the coloured gluons, which mediate the interactions between quarks, can couple to themselves. These self couplings, one believes, are responsible for quark confinement. Since the coupling strength becomes small for small separations of the quarks, one can speculate that the forces may become strong for large separations. This could explain why these fundamental constituents of matter have never been seen free in nature, and why only colour neutral hadrons are observed. A confirmation that QCD accounts for quark confinement can however only come from a non-perturbative treatment of this theory, since confinement is a consequence of the dynamics at large distances where perturbation theory breaks down.

Until 1974 all predictions of QCD were restricted to the perturbative regime. The breakthrough came with the lattice formulations of QCD by Kenneth Wilson (1974), which opened the way to the study of non-perturbative phenomena using numerical methods. By now lattice gauge theories have become a branch of particle physics in its own right, and their intimate connection to statistical mechanics make them of interest to elementary particle physicists as well as to physicists working in the latter mentioned field. Hence also those readers who are not acquainted with quantum field theory, but are working in statistical mechanics, can profit from a study of lattice gauge theories. Conversely, elementary particle physicists have profited enormously from the computational methods used in statistical mechanics, such as the high temperature expansion, cluster expansion, mean field

* for an early review see Politzer (1974).

approximation, renormalization group methods, and numerical methods.

Once the lattice formulation of QCD had been proposed by Wilson, the first question that physicists were interested in answering, was whether QCD is able to account for quark confinement. Wilson had shown that within the strong coupling approximation QCD confines quarks. As we shall see, however, this is not a justified approximation when studying the continuum limit. Numerical simulations however confirm that QCD indeed accounts for quark confinement.

There are of course many other questions that one would like to answer: does QCD account for the observed hadron spectrum? It has always been a dream of elementary particle physicists to explain why hadrons are as heavy as they are. Are there other particles predicted by QCD which have not been observed experimentally? Because of the self-couplings of the gluons, one expects that the spectrum of the Hamiltonian also contains states which are built mainly from "glue". Does QCD account for the spontaneous breakdown of chiral symmetry? It is believed that the (light) pion is the Goldstone Boson associated with a spontaneous breakdown of chiral symmetry. How do the strong interactions manifest themselves in weak decays? Can they explain the $\Delta I = 1/2$ rule in weak non-leptonic processes? How does hadronic matter behave at very high temperatures and/or high densities? Does QCD predict a phase transition to a quark gluon plasma at sufficiently high temperatures, as is expected from general theoretical considerations? This would be relevant, for example, for the understanding of the early stages of the universe.

An answer to the above mentioned questions requires a non-perturbative treatment of QCD. The lattice formulation provides the only possible framework at present to study QCD non-perturbatively.

The material in this book has been organized as follows. In the following chapter we first discuss in some detail the path integral formalism in quantum mechanics, and the path integral representation of Green functions in field theory. This formalism provides the basic framework for the lattice formulation of field theories. If the reader is well acquainted with the path integral method, he can skip all the sections of this chapter, except the last. In chapters 3 and 4 we then consider the lattice formulation of the free scalar field and the free Dirac field. While this formulation is straight-forward for the case of the scalar field, this is not the case for the Dirac field. There are several proposals that have been made in the literature for placing fermions on a space-time lattice. Of these we shall discuss in detail the Wilson and the Kogut-Susskind fermions, which have been widely used in numerical simulations, and introduce the reader to Ginsparg-

Wilson fermions, which have become of interest in more recent times, but whose implementation in numerical simulations is very time consuming. In chapters 5 and 6 we then introduce abelian and non-abelian gauge fields on the lattice, and discuss the lattice formulation of QED and QCD.

Having established the basic theoretical framework, we then present in chapter 7 a very important observable: The Wilson loop, which plays a fundamental role for studying the confinement problem. This observable will be used in chapter 8 to calculate the static potential between two charges in some simple solvable models. The purpose of that chapter is to verify in some explicit calculations that the interpretation of the Wilson loop given in chapter 7, which may have left the reader with some uneasy feelings, is correct. In chapter 9 we then discuss the continuum limit of QCD and show that this limit, which is realized at a critical point of the theory where correlations lengths diverge, corresponds to vanishing bare coupling constant. Close to the critical point the behaviour of observables as a function of the coupling constant can be determined from the renormalization group equation. Knowledge of this behaviour will be crucial for establishing whether one is extracting continuum physics in numerical simulations.

Chapter 10 is devoted to the discussion of the Michael lattice action and energy sum rules, which relate the static quark-antiquark potential to the action and energy stored in the chromoelectric and magnetic fields of a $q\bar{q}$ -pair. These sum rules are relevant for studying the energy distribution in the flux tube connecting a quark and antiquark at large separations.

Chapters 11 to 15 are devoted to various approximation schemes. Of these, the weak coupling expansion of correlation functions in lattice QCD is the most technical one. In order not to confront the reader immediately with the most complicated case, we have divided our presentation of the weak coupling expansion into three chapters. The first one deals with a simple scalar field theory and merely demonstrates the basic structure of Feynman lattice integrals. It also includes a discussion of an important theorem proved by Reisz, which is the lattice version of the well known power counting theorem for continuum Feynman integrals. In the following chapter we then increase the degree of difficulty by considering the case of lattice quantum electrodynamics (QED). Here several new concepts will be discussed, which are characteristic of a gauge theory. Readers having a fair background in the perturbative treatment of continuum QED will be able to follow easily the presentation. As an instructive application of lattice perturbation theory, we include in this chapter a 1-loop computation of the renormalization constant for the axial vector current with Wilson fermions, departing from a lat-

tice regularized Ward identity. Also included is a discussion of the ABJ-anomaly within the framework of Ginsparg-Wilson fermions. The next chapter then treats the case of QCD, which from the conceptual point of view is quite similar to the case of QED, but is technically far more involved. The Feynman rules are applied to the computation of the ABJ anomaly which is shown to be independent of the form of the lattice regularized action.

At this point we leave the analytic "terrain" and discuss in chapter 16 various algorithms that have been used in the literature to calculate observables numerically. All algorithms are based on the concept of a Markov process. We will keep the discussion very general, and only show in the last two sections of this chapter, how such algorithms are implemented in an actual calculation. Chapter 17 first summarizes some earlier numerical results obtained in the pioneering days. Because of the ever increasing computer power the numerical data becomes always more refined, and we leave it to the reader to confer the numerous proceedings for more recent results. We have however also included in this chapter some important newer developments which concern the vacuum structure of QCD and the dynamics of quark confinement.

The remaining part of the book is devoted to the study of field theories at finite temperature. It has been expected for some time that QCD undergoes a phase transition to a quark-gluon plasma, where quarks and gluons are deconfined. In chapter 18 we consider some simple bosonic and fermionic models, and discuss in detail the path-integral representation for the thermodynamical partition function. In particular we will construct such a representation for a simple fermionic system which is exact for arbitrary time step, and point out some subtle points which are not discussed in the literature. Chapter 19 is devoted to finite temperature perturbation theory in the continuum and on the lattice. The basic steps leading to the finite-temperature Feynman rules are first exemplified for a scalar field theory in the continuum. We then extend our discussion to the case of QED and QCD in the continuum as well as on the lattice and discuss in detail the temporal structure of the free propagator for naive and Wilson fermions. The Feynman rules are then applied to calculate the screening mass in QED and QCD in one-loop order, off and on the lattice. These computations will at the same time illustrate the power of frequency summation formulae, whose derivation has been relegated, in part, to two appendices.

Chapter 20 is devoted to non-perturbative aspects of QCD at finite temperature. The lattice formulation of this theory is the appropriate framework for studying the deconfinement and chiral phase transitions, and deviations of thermo-

dynamical observables from the predictions of perturbation theory at temperatures well above the phase transition. In this chapter we discuss how thermodynamical observables are computed on the lattice, and introduce an order parameter (the Wilson line or Polyakov loop) which characterizes the phases of the pure gauge theory. This order parameter plays a central role in a later section, where we present some early Monte Carlo data which gave strong support for the existence of a deconfinement phase transition. The theoretical concepts introduced in this chapter are then implemented in a simple lattice model which also serves to illustrate the power of the character expansion, a technique which is used to study $SU(N)$ gauge theories for strong coupling. The remaining part of this chapter is devoted to the high temperature phase of QCD which, as already mentioned, is expected to be that of a quark gluon plasma.

The material covered in this book should enable the reader to follow the extensive literature on this fascinating subject. What the reader will not have learned, is how much work is involved in carrying out numerical simulations. A few paragraphs in a publication will in general summarize the results obtained by several physicists over many months of very hard work. The reader will only become aware of this by speaking to physicists working in this field, or if he is involved himself in numerical calculations. Although much progress has been made in inventing new methods for calculating observables on a space time lattice, some time will still pass before one has sufficiently accurate data available to ascertain that QCD is the correct theory of strong interactions.