PREFACE TO THE THIRD EDITION

Apart from minor modifications, this new edition includes a number of topics, some of which are of great current interest. These concern in particular a discussion in chapter 17 of *instantons and calorons, and of the role played by vortices for* the confinement problem. Furthermore we have included in chapter 4 a section on Ginsparg-Wilson fermions. In chapter 10 we have added a section on the perturbative verification of the energy sum rule obtained in section 10.3. Some details of the calculations have been delegated to an appendix. New sections have also been added in chapters 14 and 15. In chapter 14 we come back to the Ginsparg-Wilson discretization of the action and discuss the ABJ anomaly within this framework. In the same chapter we also have included a detailed analysis of the renormalization of the axial vector current in one-loop order, since it provides an instructive example of how lattice regulated Ward identities can be used to determine the renormalization constants for currents. In chapter 15 we have included a very general treatment of the ABJ anomaly in QCD and show that in the continuum limit one recovers the well known result, irrespective of the precise way in which the action has been discretized.

Following our general principle which we have always tried to implement, we have done our best to convey the main ideas in a transparent way as possible, and have presented most of the non-trivial calculations in sufficient detail, so that the reader can verify them without too much effort. As always we have only included results of numerical calculations of pioneering work, be it in the early days of the lattice formulation of gauge field theories, or in more recent days.

Finally, we want to thank W. Wetzel and I. O. Stamatescu for a number of very fruitful discussions and constructive comments, and in particular Prof. Stamatescu for providing me with some unpublished plots relevant to instantons on the lattice.

Note from the author

Corrections to this book which come to the author's attention, will be posted on the World-Wide Web at

http://www.thphys.uni-heidelberg.de/~rothe_h/LGT.html

We would be grateful if the reader would inform us about any errors he may find. The e-mail address is: h.rothe@thphys.uni-heidelberg.de

PREFACE TO THE SECOND EDITION

The objective of this extended edition of the book which appeared in 1992 remains the same as at that time. The book is intended to provide the reader with the necessary theoretical background and computational tools in lattice gauge theories, to enable him to follow the vast literature on this subject, and to carry out research in this field. We have invested much effort in presenting the material in a (hopefully) transparent way. Wherever possible we exemplified complex ideas in simple models. Analytical calculations have been carried out in detail, so as to acquaint the reader with the computational techniques.

Although the numerical computations have improved substantially since the appearance of the first edition, we have refrained from including recent results in this volume. Thus apart from a new section in chapter 17, where we discuss the dual superconductor picture of confinement, the data we present is the same as in the original volume. Our emphasis is on the early pioneering work which has been the motor for subsequent investigations, and which at the same time demonstrates the difficulties that physicist were confronted with (and still are) when carrying out numerical simulations. This is in line with the introductory character of the book. For more recent results the reader should confer the numerous conference proceedings.

In this edition we have added a substantial amount of new material. In chapter 4 we have included an additional section where the fermion doubling problem is discussed in more detail. We have also added a chapter on lattice sum rules which have played an important role in the past years in numerical simulations of the flux-tube picture of confinement. Chapter 15, where we discuss the lattice Feynman rules for QCD, now also includes a derivation of the expression for the four-gluon vertex, which in the first edition had been kindly provided us by W. Wetzel. The original chapter 17 on finite temperature field theory has been expanded significantly, and is now replaced by chapters 18 and 19. Chapter 18 deals in detail with the thermodynamics of some simple, exactly solvable, bosonic and fermionic systems formulated within the path integral formalism. It provides the basis for a better understanding of the lattice formulation of field theories, and allows us to point out some subtle points which are not discussed in the literature. Chapter 19 is then devoted to finite temperature perturbation theory in the continuum and on the lattice. The first part of this chapter treats the $\lambda\phi^3$-theory in the continuum formulation, and, apart from minor changes, contains the material covered in the first edition. Thereafter we derive the finite temperature-finite chemical potential

Feynman rules for QED and QCD in the continuum and on the lattice, and apply them to calculate in detail various quantities of interest. This will provide the reader with a sound knowledge of the techniques used for carrying out perturbative computations at finite temperature and chemical potential in the continuum and on the lattice.

The final chapter is devoted to non-perturbative QCD at finite temperature. The main body of this chapter consists of the material of chapters 19 and 20 in the original version, with minor modifications. We have included an additional section, in which we implement the theoretical ideas introduced in the first two sections in a simple lattice model. This model also serves to introduce the reader to a powerful computational technique used in the literature to study lattice gauge theories for strong coupling.

What we have not discussed at all is the electro-weak sector of the standard model, and in particular lattice Higgs and Yukawa models. These models are treated in detail in the recent book by I. Montvay and G. Münster (*Quantum Fields on a Lattice*, Cambridge University Press (1994)), where the reader can also find a number of topics not covered in this book.

Writing this extended version has taken up much of the time that I should have spend with my family, and in particular with my children, who have asked me so many times in vain to play with them. I am very grateful to all of them for having had so much patience with me. I am also very grateful to T. Reisz and R. Haymaker for their critical reading of some sections of the book and their very helpful comments, and to P. Kaste for having checked a number of formulae.

PREFACE TO THE FIRST EDITION

This book is based on a one year course I held at the University of Heidelberg and on a series of lectures I gave at the "Autumn College on Techniques in Many-Body Problems" at Lahore, Pakistan, in november of 1987. These lectures have been published in the proceedings to this school by World Scientific (Rothe, 1989). I was later encouraged by the editors of World Scientific to expand on the material presented at the autumn college. This I have done in this book.

The purpose of my lectures at Lahore was to introduce lattice gauge theories to young physicists who may not have the opportunity to attend a course on this subject at their home universities. I had therefore kept the discussion as elementary as possible, including only enough thechnical details to enable the reader to follow the published literature on this subject. In this book I have expanded substantially on the material presented at Lahore, and have included a number of technical details which I felt would be very helpful to those readers who may want to carry out research in this branch of elementary particle physics. I did, however, arrange the material in such a way that those physicists who are mainly interested in getting a bird eyes view on the subject can safely skip the technical parts, without the danger of getting lost at a later stage. This concerns, in particular, the discussion in sections 4 and 5 of chapter 4 on lattice fermions, and the weak coupling expansion in lattice quantum chromodynamics (QCD), chapter 14. I have included this material for the readers convenience, since it is not discussed in such detail in the literature. I also decided to include a chapter on the path integral formalism, since the entire book is based on the path integral approach to quantization, and I do not assume that everybody is familiar with this formalism. Those readers that have never come in touch with the path integral formulation of quantum field theory may find this chapter a bit technical. However, the results we derive, of which we will make ample use in this book, are very simple, and are easily understood by everybody.

This book is mainly addressed to graduate students interested in particle physics. But it is also of interest to physicists actively engaged in research in the field of lattice gauge theories, and who may want to get a more general view on this subject. It assumes that the reader has a fair background in quantum field theory. A moderate knowledge of the continuum formulation of quantum chromodynamics would certainly be very helpful. Also physicists working in statistical mechanics may profit from reading this book, since the lattice formulation of field theories resembles closely that of complex statistical mechanical systems.

The book is divided in two parts. In the first part, comprising chapters 1 to 16, I discuss the zero temperature formulation of field theories on a space-time lattice, and in particular QCD. They are the lattice analogues of the usual continuum field theories discussed in standard text books. The second part, consisting of chapters 17 to 20, deals with finite temperature field theory. The emphasis will be on QCD, but I shall use a scalar field theory to introduce the reader to a number of new concepts which play an important role in finite temperature QCD.

Since the main goal of this book is to stimulate the readers interest in this fascinating branch of elementary particle physics, I have taken an optimistic standpoint, selecting some results of Monte Carlo calculations which illustrate the phenomena in a particularily dramatic way. I did not attempt to present a critical analysis of the results, and have left it to the reader to confer the original literature. Nor did I attempt to give a complete list of references, which the reader can find in the numerous proceedings to lattice conferences. More detailed discussions of most of the topics presented in this book can be found in the proceedings to various schools. An introduction to lattice gauge theories can also be found in the monograph by M. Creutz: *Quarks, Gluons and Lattices*, published by Cambridge University Press (1983).

Hopefully this book will stimulate some of the readers to carry out some research in the field of lattice gauge theories. If so, I have achieved the purpose it has been written for.

I like to take this opportunity to thank a number of colleages for their constructive criticisms and for having read several chapters of this book. In particular I am grateful to A. Actor, I. Bender, D. Gromes, F. Karsch, K.H. Mütter, I.O. Stamatescu and W. Wetzel. I am especially grateful to W. Wetzel for having checked a number of formulae, and for his extensive technical help in getting the manuscript into its final form. I also want to express my gratitude to Mrs. U. Einecke, and Mrs. M. Steiert for having typed so patiently the manuscript in TEX. Finally, I am particularily thankful to my family, whose continued support has made this book possible. In particular my children had to dispense of their father for many (!) hours.