Preface

Fuzzy relations appear as a natural generalization of crisp relations. While a crisp relation determines the presence or absence of interconnectedness between the elements of two or more sets, fuzzy relations supply additional information for degrees of membership, strengths of associations, interaction between elements. Advantage of fuzzy relations is also that they permit to manipulate values that can be specified in linguistic terms.

As [Zadeh and Desoer (1963)] show, in the general study of systems the relationship between input and output parameters can be modelled by fuzzy relation between input and output spaces. Investigating behavior of such systems presumes a powerful fuzzy relational calculus. The importance of the theory of fuzzy relational equations is best described by Zadeh in the preface of the monograph by [Di Nola *et al.* (1989)]: *"Human knowledge may be viewed as a collection of facts and rules, each of which may be represented as the assignment of a fuzzy relation to the unconditional or conditional possibility distribution of a variable. What this implies is that the knowledge may be viewed as a system of fuzzy relational equations. In this perspective, then, inference from a body of knowledge reduces to the solution of a system of fuzzy relational equations."*

Fuzzy relations and fuzzy relational calculus have many reasonable applications in pure and applied mathematics. The basic operations with fuzzy relations correspond to the key operations in fuzzy logic. They are implemented in all inference forward or backward chain reasoning schemes as described for instance in [Bezdek (1999), Bien and Min (1995), Dubois and Prade (2000), Dubois *et al.* (1999), Klir and Yuan (1995), Zadeh *et al.* (1996)]. The most valuable implementations are in expert systems and in artificial intelligence areas – approximate reasoning, inference systems, knowledge representation, knowledge acquisition and validation, learning, in information processing, in pattern analysis and classification, in fuzzy system science for fuzzy control and modelling, in decision making, in engineering for fault detection and diagnosis, in management, etc.

Implementing fuzziness requires developing fuzzy relational calculus, special mathematical skills, ability to operate with modern mathematics and software.

In Part 1 we propose methodology and universal algorithms for direct and inverse problem resolution in fuzzy relational calculus. They include in unified frame fuzzy linear systems of equations, fuzzy relational equations, fuzzy relational inequalities and fuzzy relational inclusions upon various compositions in bounded chain. Then intuitionistic fuzzy relational calculus is reasonably developed. Computational complexity of the problems is investigated, numerical estimations are obtained. Based on these methods and algorithms, in Part 2 we solve open problems in the theory of fuzzy machines, fuzzy languages and syntactic fuzzy pattern recognition, we propose applications in modules of expert systems. The corresponding software for fuzzy relational calculus is described in Part 3. The Appendix and CD complete the exposition.

Theoretical background is presented in *Part 1*, Chapters 1 – 6.

In *Chapter 1* we include in unified frame the recent results in fuzzy relational calculus for compositions of fuzzy relations and solving fuzzy relational equations and also outline the place of investigations in this book.

Compositions of fuzzy relations and direct problem resolution are studied in *Chapter 2*. Attention is paid on their properties, the interconnection between fuzzy relation and its representative membership matrix, as well as between compositions of fuzzy relations and matrix multiplications.

Models based on fuzzy logic require methods and algorithms for solving fuzzy relational equations. Fuzzy relational equations stay in the heart of fuzzy relational calculus. They are subject of Chapters 3 – 6.

Chapter 3 is devoted to fuzzy linear systems of equations and fuzzy relational equations over a bounded chain, when the composition is the standard one (in particular max – min). Methods and algorithms are proposed for finding the complete solution set. Solvability criterion is proved. Analytical expressions are given for determining the solutions, if the system is consistent. In case of inconsistency, the connections, that can not be satisfied simultaneously with the other connections, are marked. We also investigate algorithmical solvability and computational complexity of the problems in this subject.

Chapter 4 covers inverse problem resolution for fuzzy linear systems of inequalities and fuzzy relational inclusions. Solvability condition is proved and analytical expressions are given for the solutions. Algorithms are proposed for solving fuzzy linear systems of inequalities and fuzzy relational inclusions. Applications in fuzzy linear programming are described.

Chapters 5 and 6 are reasonable extension of the previous two chapters. We study inverse problem resolution for co-standard (in particular min – max) composition of fuzzy relational equations and for intuitionistic fuzzy relational equations thereby solving open problems in fuzzy relational calculus.

In *Chapter 5* we investigate fuzzy relational equations over a bounded chain, if the composition is the co-standard one. We solve the inverse problem developing a conventional approach, based on the methodology of Chapter 3. Specially for the min – max composition on the real closed interval [0, 1] we propose another approach, implementing duality.

In *Chapter 6* we introduce and investigate direct and inverse problems in intuitionistic fuzzy relational calculus. Their resolution is provided by the methods and algorithms developed for standard and for co-standard compositions in previous chapters.

Fuzzy relational calculus, as presented in Part 1, provides a powerful theoretical background for dealing with fuzzy machines, fuzzy languages, pattern recognition, expert systems and other artificial intelligence areas, subject of *Part 2*, Chapters 7, 8 and 9.

Several types of fuzzy finite machines are studied in *Chapter 7*, all of them over a bounded chain. We investigate fuzzy finite machines with behavior obtained upon the standard or upon the co-standard law of composition. Both classes are reasonably joined in the case of intuitionistic fuzzy finite machines. Investigation of behavior, reduction and minimization problems of all these classes of fuzzy finite machines is provided by the fuzzy algebra theory as developed in the first part of this book. We express the behavior and study various equivalences, reduction and minimization problems and their algorithmical solvability, applying direct and inverse problem resolutions, as well as the algorithms from Part 1.

In *Chapter 8* we propose how to use regular fuzzy languages for syntactic pattern recognition and classification of distorted images. We introduce intuitionistic fuzzy languages and implement them for pattern recognition and classification.

In *Chapter 9* we give engineering applications of direct and inverse problem resolution in modules of expert systems for diagnosis, testing, validation, learning, fault detection and monitoring.

Chapter 10 in *Part 3* concerns the implemented in MATLAB Fuzzy Relational Calculus Toolbox. It describes realization of functions and algorithms, as presented in Part 1, for the fuzzy algebra $\mathbb{I} = ([0,1], \max, \min)$ without any restrictions about the size of the instant and about right-hand side constants (some of them or all may be equal). Working examples are also included.

Bibliographical notes at the end of each chapter include the most substantial theoretical and applied papers and monographs and also show the place of this investigation.

The book is based on original authors' results following mainly [Peeva (2002b)] and the recent publications by Peeva and Kyosev.

Available with this book is a CD, that contains functions, described in the book, as well as a lot of examples with solutions. The software has been tested on systems of dimension 50×50, but in principal the dimension is not limited. The examples are presented with input data, complete solution set and computational solution time. The CD contains also a restricted demo version of an alternative fast algorithm (which is not described in this book) for solving fuzzy linear systems. Tests are made on a PC with CPU 900 MHz, but some of them are also tested on machines with CPU 200 MHz up to 2400 MHz. The toolbox should be of use for both teaching and research.

The toolbox is distributed under the terms of the GNU General Public License (http://www.gnu.org/copyleft/gpl.html, version 2 of the License, or any later version) as published by the Free Software Foundation.

The tools used to prepare the book are the MikTEX (http://www.miktex.org/), including its excellent YAP previewer.

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