# PREFACE

This book covers recent theoretical, numerical and experimental developments in the field of frustrated spin systems. The first edition of the book appeared in 1994 under the title " Magnetic Systems with Competing Interactions" (World Scientific) where most of works until that year have been reviewed.

The present book contains nine chapters, seven of them are new. Two chapters of the first edition have been revised with new references and comments added: these are chapter 1, Frustration - Exactly Solved Frustrated Models, and chapter 2, Properties and Phase Transition in Frustrated Spin Systems.

Frustrated spin systems have been first investigated five decades ago. Well-known examples include the Ising model on the antiferromagnetic triangular lattice studied by G. H. Wannier in 1950 and the Heisenberg helical structure discovered independently by A. Yoshimori, J. Villain and T. A. Kaplan in 1959. However, extensive investigations on frustrated spin systems have really started with the concept of frustration introduced at the same time by G. Toulouse and J. Villain in 1977 in the context of spin glasses. The frustration is generated by the competition of different kinds of interaction and/or by the lattice geometry. As a result, in the ground state all bonds are not fully satisfied. In frustrated Ising spin systems, a number of spins behave as free spins. In frustrated vector spin systems, the ground-state configuration is usually non-collinear. The ground-state of frustrated spin systems is therefore highly degenerate and new induced symmetries give rise to spectacular and often unexpected behaviors at finite temperatures.

Many properties of frustrated systems are still not well understood at present. Recent studies shown in this book reveal that established theories, numerical simulations as well as experimental techniques have encountered many difficulties in dealing with frustrated systems. In some sense, frustrated systems provide an excellent testing ground for approximations and theories.

The chapters of this book are written by researchers who have actively contributed to the field. Many results are from recent works of the authors.

The book is intended for post-graduate students as well as researchers in statistical physics, magnetism, materials science and various domains where real systems can be described with the spin language. Explicit demonstrations of formulae and full arguments leading to important results are given where it is possible to do so.

The book is organized as follows. The first two chapters deal with properties and phase transition in frustrated Ising spin systems. The two following chapters deal with the nature of the phase transition in frustrated vector spin systems. Chapters 5 and 6 treat low-dimensional frustrated quantum spin models. Chapter 7 studies the spin ice and chapter 8 shows recent experimental results on pyrochlores which are real frustrated materials. Chapter 9 deals with the classical and quantum spin glasses where both disorder and frustration act simultaneously.

I summarize in the following the contents of each chapter.

Chapter 1 shows the frustration effects in exactly solved two-dimensional Ising models. The systems considered in this chapter are periodically defined (without bond disorder). The frustration due to competing interactions will itself induce disorder in the spin orientations. After a detailed presentation of 16- and 32-vertex models, applications are made to some selected systems which possess most of the spectacular features due to the frustration such as high ground-state degeneracy, reentrance, successive phase transitions and disorder solutions. In some simple models, up to five transitions separated by two reentrant paramagnetic phases are found. A conjecture is made on the origin of the paramagnetic reentrant phase. The nature of ordering as well as the relation between the considered systems and the random-field Ising model are discussed. The relevance of disorder solutions for the reentrance phenomena is also pointed out. Evidence of the existence of partial disorder and of reentrance in complicated, non exactly solved systems is shown and discussed.

Chapter 2 deals mainly with the Ising model on the antiferromagnetic triangular and stacked triangular lattices. Ground-state properties and the nature of the phase transition are studied by various methods, as functions of the spin magnitude $S$ and nearest- and next-nearest-neighbor interactions. It is shown in this chapter that the symmetry of spin ordering strongly depends on $S$. Furthermore, due to the frustration, there exist "free" spins or "free" linear-chains, on which internal fields are cancelled out. These free spins and free linear-chains play an important role as for spin orderings. Another characteristic feature of frustrated Ising spin systems is the existence of various metastable states which is closely related to the degeneracy of ground state and also to the excited states. These metastable states may give rise to a first order phase transition as found in some models. The effects of the far-neighbor interactions on the antiferromagnetic triangular and stacked triangular lattices are clarified.

Chapter 3 is devoted to the recent advances in the renormalization group (RG) approaches to the physics of frustrated classical vector spin systems in three dimensions. The main features of the field theoretical approach to these systems including considerations on symmetries, symmetry breaking schemes, continuum limits, topological contents, ... are recalled here. An overview is provided on the phenomenological situation with emphasis put on the most striking aspect of the physics of frustrated magnets: the existence of *non universal scaling* behaviors. A review is then given on the various perturbative and non-perturbative RG approaches that have been used to investigate frustrated magnets. Finally, a large part of this chapter is devoted to a recent non-perturbative approach that has clarified the intricate physical situation of frustrated magnets.

Chapter 4 is devoted to a review on recent numerical studies dealing with frustrated vector spin systems between two and four dimensions. It is shown that various breakdowns of symmetry can occur, contrary to the case of ferromagnetic systems. The author shows that in three dimensions the transition is always of first order in the thermodynamic limit. However for "small" sizes in numerical simulations or for temperatures not "too close" to the transition temperatures in experiments, the system could display an "almost universality class" for an $O(N)/O(N-2)$ breakdown of symmetry. Many compounds studied experimentally are in this class. In two dimensions the situation is much less clear. Indeed the topological defects can play a fundamental role and their couplings with a discrete symmetry (Ising or Potts models) is not well known. In contrast to the two-dimensional case, our understanding of the three-dimensional case has increased considerably in the last decade.

Chapter 5 is devoted to a review on some theoretical advances in the field of quantum magnetism in two-dimensional systems. It is known that the spin-$\frac{1}{2}$ nearest-neighbor 2-dimensional Heisenberg models on Bravais lattices (square, triangular) are Néel ordered at $T = 0$. Frustration, small coordination number, competition between interactions can lead to specific quantum phases without magnetic long-ranged order. This long-standing subject is revived by the discovery of high-$T_c$ superconductivity in the doped cuprates and fuelled by numerous experimental studies of 2D antiferromagnetic insulators. The authors of this chapter show results on several models. They show also general properties of valence-bond crystals (VBC) and related states, as well as large-$N$ generalizations of the Heisenberg model. Some results of quantum dimer models (QDM) are presented. They provide useful insights onto the phenomenology of VBC and other systems. The authors also review some results concerning models with multiple-spin exchange and the Heisenberg model on the Kagomé lattice (and related models). Despite of an important activity on this subject, the understanding of the low-energy physics of the spin-$\frac{1}{2}$ Kagomé antiferromagnet remains a challenging problem.

Chapter 6 is devoted to an overview of some of the zero-temperature quantum spin liquid phases with unbroken SU(2) spin symmetry that have been found in one dimension. The main characteristics of these phases are discussed by means of the bosonization approach. A special emphasis is put on the interplay between frustration and quantum fluctuations in one dimension. The author presents the different spin liquid phases that occur in spin chains and spin ladders. The main effects of frustration in one-dimensional spin liquids are described. In particular, it is observed that frustration plays its trick by allowing deconfined spinons (carrying fractional $S = 1/2$ quantum number) as elementary excitations and it provides a non-trivial source of incommensurability.

Chapter 7 is devoted to the theoretical and and experimental study of so-called "spin ice", the magnetic equivalent of Pauling's model of hydrogen disorder in water ice. This represents the prototypical frustrated system, with a macroscopically degenerate ground state and extensive zero point entropy. Pauling's concept was extended by Wannier, Anderson and others to include magnetic systems and was later developed to include statistical "vertex models", some of which are exactly soluble (see Chapter 1). Spin Ice is a sixteen vertex model of "ferromagnetic frustration" that, remarkably, is found to apply to certain rare earth oxide materials. The search for a detailed microscopic understanding of these spin ice materials has cast light on many aspects of magnetic frustration and uncovered several new features of magnetic interactions (particularly the dipole interaction). For these reasons, spin ice represents an ideal laboratory in which to develop our understanding of frustrated spin systems. This chapter is a comprehensive review of the physics of spin ice including both theoretical and experimental aspects. The authors start with the concept of spin ice and its relation to the historic problem of water ice and to other frustrated systems. The following sections review the current understanding of the zero field spin ice state, the numerous field-induced states and the magnetic dynamics of the spin ice materials. Some materials related to spin ice are briefly described.

Chapter 8 describes experimental results on geometrically-frustrated magnetic systems. Much recent experimental progress has been made in the study of magnetic materials made up of antiferromagnetically-coupled magnetic moments residing on networks of corner-sharing tetrahedra. They are found in nature in a variety of cubic pyrochlore, spinel and Laves phase materials, with the magnetic moments arising from either rare earth or transition metal electrons. This review focusses on experimental progress in this area from the last ten years primarily due to neutron scattering studies. Even within this subset of three frustrated materials, quite different exotic ground states are found: the enigmatic cooperative paramagnetic ground state of $Tb_2Ti_2O_7$; the spin glass ground state of $Y_2Mo_2O_7$; and the composite spin degrees of freedom and distorted pyrochlore lattice in $ZnCr_2O_4$. It is very rewarding to appreciate the richness of the complex ground states which geometrical frustration has enabled in these real materials. There seems little doubt that this richness will continue as a theme in the elucidation of the physical properties of geometrically frustrated magnets in the near future.

Chapter 9 is devoted to a review on recent progress in spin glasses. Both the equilibrium properties and the dynamic properties are covered. The authors focus on progress in theoretical, in particular numerical, studies, while its relationship to real magnetic materials is also mentioned. A brief overview of two well-known paradigms on spin glasses is given and a summary of the predictions derived from them is presented. Then, equilibrium properties of Ising spin glass models are shown and discussed. In particular, a recent active debate, as to which paradigm is appropriate for realistic short-range spin glass models in three dimensions, is presented. Then, the dynamical properties are examined using these paradigms on a different ground, with an emphasis on aging phenomena. Some other non-equilibrium properties are also discussed. Models with continuous degrees of freedom as well as the Potts spin glass models are mentioned. The effects of weak disorder are discussed and compared to those of strong disorder. Several exact relations can be found and play an important role in shedding light on the issue under debate. Finally, results from the interplay between quantum fluctuations and randomness in spin glasses are also reviewed.

Since several problems treated in this book are currently investigated, I alert the reader that the authors of each chapter have taken the liberty to express their viewpoint on each unsettled issue.

H. T. Diep
University of Cergy-Pontoise
Cergy-Pontoise, France