

# CONTENTS

<i>Preface</i>	v
<b>1 FRUSTRATION - EXACTLY SOLVED FRUSTRATED MODELS</b>	<b>1</b>
<i>H. T. Diep and H. Giacomini</i>	
1.1 Frustration: an introduction . . . . .	1
1.1.1 Definition . . . . .	2
1.1.2 Non collinear spin configurations . . . . .	5
1.2 Frustrated Ising spin systems . . . . .	8
1.3 Mapping between Ising models and vertex models . . . . .	10
1.3.1 The 16-vertex model . . . . .	10
1.3.2 The 32-vertex model . . . . .	12
1.3.3 Disorder solutions for two-dimensional Ising models	23
1.4 Reentrance in exactly solved frustrated Ising spin systems .	26
1.4.1 Centered square lattice . . . . .	27
1.4.1.1 Phase diagram . . . . .	28
1.4.1.2 Nature of ordering and disorder solutions .	28
1.4.2 Kagomé lattice . . . . .	31
1.4.2.1 Model with nn and nnn interactions . . . . .	31
1.4.2.2 Generalized Kagomé lattice . . . . .	31
1.4.3 Centered honeycomb lattice . . . . .	37
1.4.4 Periodically dilute centered square lattices . . . . .	40
1.4.4.1 Model with three centers . . . . .	44
1.4.4.2 Model with two adjacent centers . . . . .	46
1.4.4.3 Model with one center . . . . .	47
1.4.5 Random-field aspects of the models . . . . .	48

1.5	Evidence of partial disorder and reentrance in other frustrated systems . . . . .	50
1.6	Conclusion . . . . .	54
	Acknowledgements . . . . .	57
	References . . . . .	57
<b>2</b>	<b>PROPERTIES AND PHASE TRANSITIONS IN FRUSTRATED ISING SYSTEMS</b>	<b>59</b>
	<i>Ojiro Nagai, Tsuyoshi Horiguchi and Seiji Miyashita</i>	
2.1	Introduction . . . . .	59
2.2	Ising model on two-dimensional frustrated lattice and on stacked frustrated lattice . . . . .	62
2.3	Ising model on antiferromagnetic triangular lattice . . . . .	65
2.4	Ising model on stacked antiferromagnetic triangular lattice . . . . .	71
2.5	Ising model with large $S$ on antiferromagnetic triangular lattice . . . . .	76
2.6	Ising model with infinite-spin on antiferromagnetic triangular lattice . . . . .	80
2.7	Ising-like Heisenberg model on antiferromagnetic triangular lattice . . . . .	82
2.8	Ising model with infinite-spin on stacked antiferromagnetic triangular lattice . . . . .	82
2.9	Phase diagram in spin-magnitude versus temperature for Ising models with spin $S$ on stacked antiferromagnetic triangular lattice . . . . .	87
2.10	Effect of antiferromagnetic interaction between next-nearest-neighbor spins in $xy$ -plane . . . . .	90
2.11	Three-dimensional Ising paramagnet . . . . .	96
2.12	Concluding remarks . . . . .	102
	Acknowledgements . . . . .	103
	References . . . . .	103
<b>3</b>	<b>RENORMALIZATION GROUP APPROACHES TO FRUSTRATED MAGNETS IN D=3</b>	<b>107</b>
	<i>B. Delamotte, D. Mouhanna and M. Tissier</i>	
3.1	Introduction . . . . .	107

3.2	The STA model and generalization . . . . .	109
3.2.1	The lattice model, its continuum limit and symmetries . . . . .	109
3.2.2	The Heisenberg case . . . . .	112
3.2.3	The XY case . . . . .	115
3.2.4	Generalization . . . . .	116
3.3	Experimental and numerical situations . . . . .	116
3.3.1	The XY systems . . . . .	116
3.3.1.1	The experimental situation . . . . .	117
3.3.1.2	The numerical situation . . . . .	119
3.3.1.3	Summary . . . . .	121
3.3.2	The Heisenberg systems . . . . .	122
3.3.2.1	The experimental situation . . . . .	122
3.3.2.2	The numerical situation . . . . .	124
3.3.2.3	Summary . . . . .	126
3.3.3	The $N = 6$ STA . . . . .	126
3.3.4	Conclusion . . . . .	127
3.4	A brief chronological survey of the theoretical approaches .	127
3.5	The perturbative situation . . . . .	131
3.5.1	The Nonlinear Sigma ( $NL\sigma$ ) model approach . . . .	131
3.5.2	The Ginzburg-Landau-Wilson (GLW) model approach . . . . .	136
3.5.2.1	The RG flow . . . . .	136
3.5.2.2	The three and five-loop results in $d = 4 - \epsilon$ .	137
3.5.2.3	The improved three and five-loop results . .	137
3.5.2.4	The three-loop results in $d = 3$ . . . . .	138
3.5.2.5	The large- $N$ results . . . . .	138
3.5.2.6	The six-loop results in $d = 3$ . . . . .	139
3.5.3	The six-loop results in $d = 3$ re-examined . . . . .	141
3.5.3.1	Conclusion . . . . .	143
3.6	The effective average action method . . . . .	143
3.6.1	The effective average action equation . . . . .	143
3.6.2	Properties . . . . .	148
3.6.3	Truncations . . . . .	149
3.6.4	Principle of the calculation . . . . .	150
3.6.5	The $O(N) \times O(2)$ model . . . . .	151
3.6.5.1	The flow equations . . . . .	153

3.6.6	Tests of the method and first results . . . . .	154
3.6.7	The physics in $d = 3$ according to the NPRG approach . . . . .	158
3.6.7.1	The physics in $d = 3$ just below $N_c(d)$ : Scaling with a pseudo-fixed point and minimum of the flow . . . . .	159
3.6.7.2	Scaling with or without pseudo-fixed point: The Heisenberg and XY cases . . . . .	161
3.6.7.3	The integration of the RG flow . . . . .	164
3.6.7.4	The Heisenberg case . . . . .	165
3.6.7.5	The XY case . . . . .	167
3.6.8	Conclusion . . . . .	169
3.7	Conclusion and prospects . . . . .	169
	Acknowledgements . . . . .	171
	References . . . . .	171
<b>4</b>	<b>PHASE TRANSITIONS IN FRUSTRATED VECTOR SPIN SYSTEMS: NUMERICAL STUDIES</b>	<b>177</b>
	<i>D. Loison</i>	
	Abbreviations . . . . .	177
4.1	Introduction . . . . .	177
4.2	Breakdown of symmetry . . . . .	178
4.2.1	Symmetry in the high-temperature region . . . . .	179
4.2.2	Breakdown of symmetry for ferromagnetic systems .	179
4.2.3	Breakdown of symmetry for frustrated systems . . .	181
4.2.3.1	Stacked triangular antiferromagnetic lattices	181
4.2.3.2	bct helimagnets . . . . .	184
4.2.3.3	Stacked $J_1$ - $J_2$ square lattices . . . . .	185
4.2.3.4	The simple cubic $J_1$ - $J_2$ lattice . . . . .	185
4.2.3.5	$J_1$ - $J_2$ - $J_3$ lattice . . . . .	186
4.2.3.6	Villain lattice and fully frustrated simple cubic lattice . . . . .	186
4.2.3.7	Face-centered cubic lattice (fcc) . . . . .	187
4.2.3.8	Hexagonal-close-packed lattice (hcp) . . . . .	187
4.2.3.9	Pyrochlores . . . . .	188
4.2.3.10	Other lattices . . . . .	188
4.2.3.11	STAR lattices . . . . .	188

4.2.3.12	Dihedral lattices $V_{N,2}$ . . . . .	189
4.2.3.13	Right-handed trihedral lattices $V_{3,3}$ . . . . .	189
4.2.3.14	P-hedral lattices $V_{N,P}$ . . . . .	190
4.2.3.15	Ising and Potts- $V_{N,1}$ model . . . . .	190
4.2.3.16	Ising and Potts- $V_{N,2}$ model . . . . .	191
4.2.3.17	Landau-Ginzburg model . . . . .	191
4.2.3.18	Cubic term in Hamiltonian . . . . .	191
4.2.3.19	Summary . . . . .	192
4.3	Phase transitions between two and four dimensions: $2 < d \leq 4$ . . . . .	192
4.3.1	$O(N)/O(N-2)$ breakdown of symmetry . . . . .	193
4.3.1.1	Fixed points . . . . .	193
4.3.1.2	MCRG and first-order transition . . . . .	195
4.3.1.3	Complex fixed point or minimum in the flow . . . . .	196
4.3.1.4	Experiment . . . . .	200
4.3.1.5	Value of $N_c$ . . . . .	201
4.3.1.6	Phase diagram $(N, d)$ . . . . .	202
4.3.1.7	Renormalization-group expansions . . . . .	202
4.3.1.8	Short historical review . . . . .	204
4.3.1.9	Relations with the Potts model . . . . .	205
4.3.2	$O(N)/O(N-P)$ breakdown of symmetry for $d = 3$ . . . . .	206
4.3.3	$Z_2 \otimes SO(N)/SO(N-1)$ breakdown of symmetry for $d = 3$ . . . . .	207
4.3.4	$Z_3 \otimes SO(N)/SO(N-1)$ breakdown of symmetry for $d = 3$ . . . . .	208
4.3.5	$Z_q \otimes O(N)/O(N-2)$ and other breakdown of symmetry in $d = 3$ . . . . .	208
4.4	Conclusion . . . . .	209
4.5	$O(N)$ frustrated vector spins in $d = 2$ . . . . .	210
4.5.1	Introduction . . . . .	210
4.5.2	Non frustrated $XY$ spin systems . . . . .	210
4.5.3	Frustrated $XY$ spin systems: $Z_2 \otimes SO(2)$ . . . . .	210
4.5.4	Frustrated $XY$ spin systems: $Z_3 \otimes SO(2)$ . . . . .	213
4.5.5	Frustrated $XY$ spin systems: $Z_2 \otimes Z_2 \otimes SO(2)$ and $Z_3 \otimes Z_2 \otimes SO(2)$ . . . . .	214
4.5.6	Frustrated Heisenberg spin systems: $SO(3)$ . . . . .	214

4.5.7	Frustrated Heisenberg spin systems: $Z_2 \otimes SO(3)$ , $Z_3 \otimes SO(3)$ . . . . .	215
4.5.8	Topological defects for $N \geq 4$ . . . . .	216
4.6	General conclusions . . . . .	216
	Acknowledgements . . . . .	216
	Appendix A: Monte Carlo simulation . . . . .	216
	Appendix B: Renormalization group . . . . .	220
	References . . . . .	223
<b>5</b>	<b>TWO-DIMENSIONAL QUANTUM ANTIFERROMAGNETS</b> . . . . .	<b>229</b>
	<i>Grégoire Misguich and Claire Lhuillier</i>	
5.1	Introduction . . . . .	229
5.2	$J_1$ - $J_2$ model on the square lattice . . . . .	231
5.2.1	Classical ground-state and spin-wave analysis . . . . .	231
5.2.2	Order by disorder ( $J_2 > J_1/2$ ) . . . . .	232
5.2.3	Non-magnetic region ( $J_2 \simeq J_1/2$ ) . . . . .	233
	5.2.3.1 Series expansions . . . . .	234
	5.2.3.2 Exact diagonalizations . . . . .	236
	5.2.3.3 Quantum Monte Carlo . . . . .	236
5.3	Valence-bond crystals . . . . .	238
5.3.1	Definitions . . . . .	238
5.3.2	One-dimensional and quasi one-dimensional examples (spin- $\frac{1}{2}$ systems) . . . . .	239
5.3.3	Valence bond solids . . . . .	240
5.3.4	Two-dimensional examples of VBC . . . . .	241
	5.3.4.1 Without spontaneous lattice symmetry breaking . . . . .	241
	5.3.4.2 With spontaneous lattice symmetry breaking . . . . .	243
5.3.5	Methods . . . . .	245
5.3.6	Summary of the properties of VBC phases . . . . .	246
5.4	Large- $N$ methods . . . . .	247
5.4.1	Bond variables . . . . .	248
5.4.2	$SU(N)$ . . . . .	249
5.4.3	$Sp(N)$ . . . . .	249
	5.4.3.1 Gauge invariance . . . . .	251

5.4.3.2	Mean-field ( $N = \infty$ limit) . . . . .	251
5.4.3.3	Fluctuations about the mean-field solution . . . . .	252
5.4.3.4	Topological effects - Instantons and spontaneous dimerization . . . . .	254
5.4.3.5	Deconfined phases . . . . .	255
5.5	Quantum dimer models . . . . .	256
5.5.1	Hamiltonian . . . . .	256
5.5.2	Relation with spin- $\frac{1}{2}$ models . . . . .	257
5.5.3	Square lattice . . . . .	259
5.5.3.1	Transition graphs and topological sectors . . . . .	259
5.5.3.2	Staggered VBC for $V/J > 1$ . . . . .	260
5.5.3.3	Columnar crystal for $V < 0$ . . . . .	260
5.5.3.4	Plaquette phase . . . . .	261
5.5.3.5	Rokhsar-Kivelson point . . . . .	261
5.5.4	Hexagonal lattice . . . . .	262
5.5.5	Triangular lattice . . . . .	263
5.5.5.1	RVB liquid at the RK point . . . . .	264
5.5.5.2	Topological order . . . . .	265
5.5.6	Solvable QDM on the Kagomé lattice . . . . .	266
5.5.6.1	Hamiltonian . . . . .	266
5.5.6.2	RK ground-state . . . . .	266
5.5.6.3	Ising pseudo-spin variables . . . . .	268
5.5.6.4	Dimer-dimer correlations . . . . .	269
5.5.6.5	Visons excitations . . . . .	269
5.5.6.6	Spinons deconfinement . . . . .	271
5.5.6.7	$\mathbb{Z}_2$ gauge theory . . . . .	272
5.5.7	A QDM with an extensive ground-state entropy . . . . .	273
5.6	Multiple-spin exchange models . . . . .	274
5.6.1	Physical realizations of multiple-spin interactions . . . . .	274
5.6.1.1	Nuclear magnetism of solid $^3\text{He}$ . . . . .	274
5.6.1.2	Wigner crystal . . . . .	276
5.6.1.3	Cuprates . . . . .	276
5.6.2	Two-leg ladders . . . . .	276
5.6.3	MSE model on the square lattice . . . . .	278
5.6.4	RVB phase of the triangular $J_2$ - $J_4$ MSE . . . . .	278
5.6.4.1	Non-planar classical ground-states . . . . .	279
5.6.4.2	Absence of Néel LRO . . . . .	279

5.6.4.3	Local singlet-singlet correlations - Absence of lattice symmetry breaking . . . . .	280
5.6.4.4	Topological degeneracy and Lieb-Schultz-Mattis theorem . . . . .	281
5.6.4.5	Deconfined spinons . . . . .	282
5.6.5	Other models with MSE interactions . . . . .	283
5.7	Antiferromagnets on the Kagomé (and related) lattices . . . . .	284
5.7.1	Miscellaneous models on the kagome lattice . . . . .	284
5.7.2	Spin- $\frac{1}{2}$ Heisenberg model on the Kagomé lattice: An extreme play-ground for “quantum fluctuations” . . . . .	285
5.7.2.1	Ground-state energy per spin . . . . .	285
5.7.2.2	Correlations . . . . .	286
5.7.2.3	Spin-gap . . . . .	286
5.7.2.4	An exceptional density of low lying excitations in the singlet sector . . . . .	287
5.7.2.5	Absence of gap in the singlet sector . . . . .	290
5.7.2.6	Anomalous density of states in other spin sectors . . . . .	291
5.7.3	Nearest-neighbor RVB description of the spin- $\frac{1}{2}$ Kagomé antiferromagnet . . . . .	292
5.7.4	Experiments in compounds with Kagomé-like lattices . . . . .	293
5.7.5	“Haldane’s conjecture” . . . . .	293
5.8	Conclusions . . . . .	294
	Acknowledgements . . . . .	296
	References . . . . .	296
<b>6</b>	<b>ONE-DIMENSIONAL SPIN LIQUIDS</b>	<b>307</b>
	<i>P. Lecheminant</i>	
6.1	Introduction . . . . .	307
6.2	Unfrustrated spin chains . . . . .	310
6.2.1	Spin- $\frac{1}{2}$ Heisenberg chain . . . . .	310
6.2.2	Haldane’s conjecture . . . . .	313
6.2.3	Haldane spin liquid: Spin-1 Heisenberg chain . . . . .	315
6.2.4	General spin- $S$ case . . . . .	319
6.2.5	Two-leg spin ladder . . . . .	321
6.2.6	Non-Haldane spin liquid . . . . .	327
6.3	Frustration effects . . . . .	331

6.3.1	Semiclassical analysis . . . . .	331
6.3.2	Spin liquid phase with massive deconfined spinons . . . . .	334
6.3.3	Field theory of spin liquid with incommensurate correlations . . . . .	341
6.3.4	Extended criticality stabilized by frustration . . . . .	345
6.3.4.1	Critical phases with SU(N) quantum criticality . . . . .	346
6.3.4.2	Chirally stabilized critical spin liquid . . . . .	350
6.4	Concluding remarks . . . . .	353
	Acknowledgements . . . . .	356
	References . . . . .	356
<b>7</b>	<b>SPIN ICE</b> . . . . .	<b>367</b>
	<i>Steven T. Bramwell, Michel J. P. Gingras and Peter C. W. Holdsworth</i>	
7.1	Introduction . . . . .	368
7.2	From water ice to spin ice . . . . .	371
7.2.1	Pauling's model . . . . .	371
7.2.2	Why is the zero point entropy not zero? . . . . .	373
7.2.3	Generalizations of Pauling's model . . . . .	374
7.2.3.1	Wannier's model . . . . .	374
7.2.3.2	Anderson's model . . . . .	375
7.2.3.3	Vertex models . . . . .	376
7.2.3.4	Possibility of realizing magnetic vertex models . . . . .	376
7.2.4	Spin ice . . . . .	378
7.2.4.1	Definition of the spin ice model and its application to $\text{Ho}_2\text{Ti}_2\text{O}_7$ . . . . .	378
7.2.4.2	Identification of spin ice materials . . . . .	380
7.2.4.3	Basic properties of the spin ice materials . . . . .	380
7.2.5	Spin ice as a frustrated magnet . . . . .	383
7.2.5.1	Frustration and underconstraining . . . . .	383
7.2.5.2	$\langle 111 \rangle$ pyrochlore models . . . . .	384
7.3	Properties of the zero field spin ice state . . . . .	385
7.3.1	Experimental properties . . . . .	385
7.3.1.1	Heat capacity: Zero point entropy . . . . .	385

7.3.1.2	Low field magnetic susceptibility: Spin freezing . . . . .	389
7.3.1.3	Spin arrangement observed by neutron scattering . . . . .	390
7.3.2	Microscopic theories and experimental tests . . . . .	391
7.3.2.1	Near-neighbour spin ice model: Successes and failures . . . . .	391
7.3.2.2	The problem of treating the dipolar interaction . . . . .	393
7.3.2.3	The Ewald Monte Carlo . . . . .	397
7.3.2.4	Mean-field theory . . . . .	401
7.3.2.5	The loop Monte Carlo . . . . .	404
7.3.2.6	Application of the dipolar model to neutron scattering results . . . . .	413
7.3.2.7	How realistic is the dipolar model? . . . . .	413
7.4	Field-induced phases . . . . .	414
7.4.1	Theory . . . . .	415
7.4.1.1	Near neighbour model . . . . .	415
7.4.1.2	Dipolar model . . . . .	418
7.4.2	Magnetization measurements above $T = 1$ K . . . . .	418
7.4.3	Bulk measurements at low temperature . . . . .	419
7.4.3.1	[111] direction . . . . .	419
7.4.3.2	[110] direction . . . . .	423
7.4.3.3	[100] direction . . . . .	423
7.4.3.4	[211] direction . . . . .	424
7.4.3.5	Powder measurements . . . . .	425
7.4.4	Neutron scattering results . . . . .	426
7.4.4.1	[110] direction . . . . .	426
7.4.4.2	[100], [111] and [211] directions . . . . .	428
7.4.5	Kagomé ice . . . . .	428
7.4.5.1	Basic Kagomé ice model and mappings . . . . .	429
7.4.5.2	Experimental results: Specific heat . . . . .	431
7.4.5.3	Theory of the Kagomé ice state: Kastelyn transition . . . . .	432
7.5	Spin dynamics of the spin ice materials . . . . .	433
7.5.1	Experimental quantities of interest . . . . .	433

7.5.1.1	Correlation functions and neutron scattering . . . . .	433
7.5.1.2	Fluctuation-dissipation theorem and AC-susceptibility . . . . .	434
7.5.1.3	Spectral shape function . . . . .	434
7.5.1.4	Exponential relaxation . . . . .	435
7.5.2	Differences between $\text{Ho}_2\text{Ti}_2\text{O}_7$ and $\text{Dy}_2\text{Ti}_2\text{O}_7$ . . . . .	436
7.5.3	Relaxation at high temperature, $T \sim 15$ K and above . . . . .	436
7.5.3.1	AC-susceptibility (AC- $\chi$ ) . . . . .	436
7.5.3.2	Neutron Spin Echo (NSE) . . . . .	436
7.5.3.3	Origin of the 15 K AC-susceptibility peak . . . . .	439
7.5.4	Relaxation in the range $1 \text{ K} \leq T \leq 15 \text{ K}$ . . . . .	441
7.5.4.1	AC-susceptibility: Phenomenological model . . . . .	441
7.5.4.2	AC-susceptibility: Towards a microscopic model . . . . .	441
7.5.5	Spin dynamics in the spin ice regime below 1 K . . . . .	443
7.5.5.1	Slow relaxation . . . . .	443
7.5.5.2	Evidence for residual dynamics in the frozen state . . . . .	444
7.5.6	Doped spin ice . . . . .	445
7.5.7	Spin ice under pressure . . . . .	446
7.6	Spin ice related materials . . . . .	446
7.6.1	Rare earth titanates . . . . .	447
7.6.2	Other pyrochlores related to spin ice . . . . .	448
7.7	Conclusions . . . . .	449
	Acknowledgements . . . . .	450
	References . . . . .	451
<b>8</b>	<b>EXPERIMENTAL STUDIES OF FRUSTRATED PYROCHLORE ANTIFERROMAGNETS</b>	<b>457</b>
	<i>Bruce D. Gaulin and Jason S. Gardner</i>	
8.1	Introduction . . . . .	458
8.2	Pyrochlore lattices . . . . .	459
8.3	Neutron scattering techniques . . . . .	461
8.4	Cooperative paramagnetism in $\text{Tb}_2\text{Ti}_2\text{O}_7$ . . . . .	463
8.5	The spin glass ground state in $\text{Y}_2\text{Mo}_2\text{O}_7$ . . . . .	474

8.6	Composite spin degrees of freedom and spin-Peierls-like ground state in the frustrated spinel $\text{ZnCr}_2\text{O}_4$ . . . . .	483
8.7	Conclusions and outlook . . . . .	486
	Acknowledgements . . . . .	486
	References . . . . .	487
<b>9</b>	<b>RECENT PROGRESS IN SPIN GLASSES</b> . . . . .	<b>491</b>
	<i>N. Kawashima and H. Rieger</i>	
9.1	Two pictures . . . . .	492
9.1.1	Mean-field picture . . . . .	493
9.1.2	Droplet picture . . . . .	496
9.2	Equilibrium properties of two-dimensional Ising spin glasses . . . . .	498
9.2.1	Zero-temperature transition? . . . . .	498
9.2.2	Droplet argument for Gaussian-coupling models . . . . .	500
9.2.3	Droplets in Gaussian-coupling models: Numerics . . . . .	500
9.2.4	Finite-temperature transition? . . . . .	503
9.3	Equilibrium properties of three-dimensional models . . . . .	503
9.3.1	Finite temperature transition? . . . . .	504
9.3.2	Universality class . . . . .	505
9.3.3	Low-temperature phase of the $\pm J$ model . . . . .	507
9.3.4	Low-temperature phase of the Gaussian-coupling model . . . . .	511
9.3.5	Effect of magnetic fields . . . . .	517
9.3.6	Sponge-like excitations . . . . .	518
9.3.7	TNT picture — Introduction of a new scaling length . . . . .	519
9.3.8	Arguments supporting the droplet picture . . . . .	520
9.4	Models in four or higher dimensions . . . . .	521
9.5	Aging . . . . .	523
9.5.1	A growing length scale during aging? . . . . .	523
9.5.2	Two time quantities: Isothermal aging . . . . .	528
9.5.3	More complicated temperature protocols . . . . .	533
9.5.4	Violation of the fluctuation-dissipation theorem . . . . .	538
9.5.5	Hysteresis in spin glasses . . . . .	541
9.6	Equilibrium properties of classical $XY$ and Heisenberg spin glasses . . . . .	544

9.6.1	Continuous spin models in three dimensions . . . . .	544
9.6.2	Continuous spin models in higher dimensions . . . . .	549
9.6.3	Potts spin glasses . . . . .	550
9.7	Weak disorder . . . . .	551
9.7.1	Phase diagram of the discrete spin models . . . . .	552
9.7.2	Dynamical properties . . . . .	554
9.7.3	The renormalization group approach for the discrete models . . . . .	555
9.7.4	The location of the multi-critical point . . . . .	558
9.7.5	Phase diagram of the random $XY$ model in two dimensions . . . . .	560
9.8	Quantum spin glasses . . . . .	562
9.8.1	Random transverse Ising models . . . . .	563
9.8.2	Mean-field theory . . . . .	571
9.8.3	Mean-field theory — Dissipative effects . . . . .	575
9.8.4	Mean-field theory — Dynamics . . . . .	579
9.8.5	Heisenberg quantum spin glasses . . . . .	581
	9.8.5.1 Finite dimensions . . . . .	582
	9.8.5.2 Mean-field model . . . . .	582
9.9	Summary and remaining problems . . . . .	584
	Acknowledgements . . . . .	586
	References . . . . .	587