

Contents

Preface.....	vii
1. The Triangular Distribution.....	1
1.1 An Historical Overview.....	1
1.2 Deriving the CDF utilizing a Geometric Argument.....	6
1.3 Moments of Triangular Distributions.....	8
1.4 Maximum Likelihood Method for the Threshold Parameter θ	11
1.4.1 <i>An illustrative example</i>	14
1.5 Three Parameter Maximum Likelihood Estimation.....	16
1.5.1 <i>Some details about the functions $G(a, b)$ and $\hat{r}(a, b)$</i>	20
1.5.2 <i>ML estimation procedure in pseudo Pascal</i>	25
1.5.2.1 <i>The search routine Bsearch</i>	26
1.5.2.2 <i>The search routine ABSearch</i>	27
1.6 Solving for a and b using a Lower and Upper Quantile Estimate.....	28
1.7 Concluding Remarks.....	31
2. Some Early Extensions of the Triangular Distribution.....	33
2.1 The Topp and Leone Distribution.....	33
2.1.1 <i>Location estimates of Topp and Leone distributions</i>	38
2.1.2 <i>Variance of Topp and Leone distributions</i>	42
2.1.3 <i>Maximum likelihood estimation</i>	43
2.2 The Trapezoidal Distribution.....	44
2.2.1 <i>Moments of the trapezoidal distribution</i>	46
2.2.2 <i>Inverse cumulative distribution function</i>	49
2.3 A Linear Combination of Uniform Variables.....	49
2.3.1 <i>Elementary derivation of the cdf of X given by (2.34)</i>	54
2.3.2 <i>Algorithm for evaluating the cdf given by (2.36)</i>	61
2.4 Concluding Remarks.....	62

3. The Standard Two-Sided Power Distribution.....	63
3.1 Introduction: The Leading Example.....	63
3.2 The Standard Two-Sided Power Distributions.....	71
3.2.1 Moments.....	73
3.2.2 Properties of the cdf.....	74
3.2.3 Quantile properties.....	75
3.2.4 Some limiting distributions.....	76
3.2.5 Relative entropy.....	78
3.3 Maximum Likelihood Method of Estimating Parameters.....	79
3.3.1 An illustrative example.....	84
3.4 Method of Moments.....	86
3.5 Moment Ratio Diagram Comparison with the Beta Family.....	90
3.6 Musings on STSP and Beta Families and Concluding Remarks.....	94
4. The Two-Sided Power Distribution.....	97
4.1 Introduction: The Four-Parameter TSP Distribution.....	97
4.2 Four-Parameter Maximum Likelihood Estimation.....	100
4.2.1 An illustrative example.....	104
4.3 Elicitation Methods for the TSP Distributions.....	109
4.3.1 Indirect elicitation of the shape parameter n	112
4.3.2 The PERT "controversy" via an illustrative example.....	113
4.3.2.1 Sources for the PERT controversy in Sec. 4.3.2.....	116
4.3.2.2 Attempts to improve the mean and variance estimation using expert judgment.....	121
4.3.3 Indirect elicitation of the parameters a , b and n	123
4.3.3.1 Description of the numerical algorithm.....	127
4.3.3.2 Assessment of the effect of the elicitation procedure in a PERT example.....	132
4.3.3.3 Some mathematical details regarding the algorithm in Sec. 4.3.3.1.....	136
4.4 Concluding Remarks.....	143

- 5. The Generalized Trapezoidal Distribution..... 147**
 - 5.1 Illustrative Example..... 147
 - 5.2 The Functional Form of the Generalized Trapezoidal Density.... 150
 - 5.2.1 Construction of the probability density function..... 153
 - 5.2.2 Mixing behavior of the component density functions..... 157
 - 5.3 Basic Properties of the Generalized Trapezoidal Distribution.... 159
 - 5.3.1 Cumulative distribution function..... 159
 - 5.3.2 Moments..... 159
 - 5.4 Concluding Remarks..... 162

- 6. Uneven Two-Sided Power Distributions..... 163**
 - 6.1 Motivation..... 164
 - 6.2 Derivation of UTSP Family by a Single Limiting Operation..... 165
 - 6.3 Some Properties of the Uneven STSP Distribution..... 168
 - 6.4 ML Estimation Procedure for USTSP Distributions..... 173
 - 6.4.1 Mathematical details of the ML estimation procedure..... 176
 - 6.4.1.1 Step 1: maximizing over the LHS power parameter n_1 177
 - 6.4.1.2 Step 2: maximizing over the RHS power parameter n_3 178
 - 6.4.1.3 Step 3: maximizing over the jump parameter α 179
 - 6.4.1.4 Step 4: maximizing over the threshold parameter θ 180
 - 6.5 Illustrative Example..... 183

- 7. The Reflected Generalized Topp and Leone Distribution..... 195**
 - 7.1 Introduction..... 196
 - 7.2 Cumulative Distribution Function and Density Function..... 198
 - 7.3 Properties of Standard RGTL Distributions..... 201
 - 7.3.1 Limiting distributions..... 201
 - 7.3.2 Stochastic dominance properties..... 202
 - 7.3.3 Mode analysis of standard RGTL distributions..... 203
 - 7.3.4 Failure rate function..... 206
 - 7.3.5 Cumulative moments..... 206
 - 7.3.6 Inverse cumulative distribution function..... 210
 - 7.4 Maximum Likelihood Estimation of SRGTL Parameters..... 211
 - 7.5 Fitting 2001 US Household Income Distribution Data..... 213

8. A Generalized Framework for Two-Sided Distributions.....	225
8.1. Standard Two-Sided Families of Distributions.....	225
8.2. The Two-Sided (Linear) Slope Distribution.....	232
8.2.1 <i>Moment estimation for ST-SS distributions.....</i>	238
8.2.2. <i>Maximum likelihood estimation of ST-SS parameters.....</i>	241
Epilogue.....	249
Appendix A: Graphical Overview of Continuous Univariate Families of Distributions possessing a Bounded Domain.....	251
Appendix B: The Johnson S_B Distribution.....	255
B.1. Motivation and Representation.....	255
B.2. Some Properties of the Johnson S_B Family.....	257
B.2.1 <i>Median value.....</i>	259
B.2.2 <i>Mode analysis.....</i>	260
B.2.3 <i>Moments and parameter estimation.....</i>	263
B.2.4 <i>Some thoughts on limiting distributions.....</i>	265
B.3 Concluding Remarks.....	267
Bibliography.....	249
Author Index.....	279
Subject Index.....	283