Preface

This book discusses the properties of microscopic particles in nonlinear systems, principles of the nonlinear quantum mechanical theory, and its applications in condensed matter, polymers, and biological systems. It is intended for researchers, graduate students, and upper level undergraduate students.

About the Book

Some materials in the book are based on the lecture notes for a graduate course "Problems in nonlinear quantum theory" given by one of the authors (X. F. Pang) in his university in the 1980s, and a book entitled "Theory of Nonlinear Quantum Mechanics" (in Chinese) by the same author in 1994. However, the contents were completely rewritten in this English edition, and in the process, we incorporated recent results related to the nonlinear Schrödinger equations and the nonlinear Klein-Gordon equations based on research of the authors as well as other scientists in the field.

The following topics are covered in 10 chapters in this book, the necessity for constructing a nonlinear quantum mechanical theory; the theoretical and experimental foundations on which the nonlinear quantum mechanical theory is based; the elementary principles and the theory of nonlinear quantum mechanics; the wave-corpuscle duality of particles in the theory; nonlinear interaction and localization of particles; the relations between nonlinear and linear quantum theories; the properties of nonlinear quantum mechanics, including simultaneous determination of position and momentum of particles, self-consistence and completeness of the theory; methods of solving nonlinear quantum mechanical problems; properties of particles in various nonlinear systems and applications to exciton, phonon, polaron, electron, magnon and proton in physical, biological and polymeric systems. In particular, an in-depth discussion on the wave-corpuscle duality of microscopic particles in nonlinear systems is given in this book.

The book is organized as follows. We start with a brief review on the postulates of linear quantum mechanics, its successes and problems encountered by the linear quantum mechanics in Chapter 1. In Chapter 2, we discuss some macroscopic quantum effects which form the experimental foundation for a new nonlinear quantum theory, and the properties of microscopic particles in the macroscopic quantum systems which provide a theoretical base for the establishment of the nonlinear quantum theory. The fundamental principles on which the new theory is based and the theory of nonlinear quantum mechanics as proposed by Pang *et al.* are given in Chapter 3. The close relations among the properties of macroscopic quantum effects; nonlinear interactions and soliton motions of microscopic particles in macroscopic quantum systems play an essential role in the establishment of this theory. In Chapter 4, we examine in details the wave-corpuscle duality of particles in nonlinear systems. In Chapter 5, we look into the mechanisms of nonlinear interactions and their relations to localization of particles. In the next chapter, features of the nonlinear and linear quantum mechanical theories are compared; the self-consistence and completeness of the theory were examined; and finally solutions and properties of the time-independent nonlinear quantum mechanical equations, and their relations to the original quantum mechanics are discussed. We will show that problems existed in the original quantum mechanics can be explained by the new nonlinear quantum mechanical theory. Chapter 7 shows the methods of solving various kinds of nonlinear quantum mechanical problems. The dynamic properties of microscopic particles in different nonlinear systems are discussed in Chapter 8. Finally in Chapters 9 and 10, applications of the theory to exciton, phonon, electron, polaron, proton and magnon in various physical systems, such as condensed matter, polymers, molecules and living systems, are explored.

The book is essentially composed of three parts. The first part consists of Chapters 1 and 2, gives a review on the linear quantum mechanics, and the important experimental and theoretical studies that lead to the establishment of the nonlinear quantum-mechanical theory. The nonlinear theory of quantum mechanics itself as well as its essential features are described in second part (Chapters 3-8). In the third part (Chapters 9 and 10), we look into applications of this theory in physics, biology and polymer, *etc.*

An Overview

Nonlinear quantum mechanics (NLQM) is a theory for studying properties and motion of microscopic particles (MIPs) in nonlinear systems which exhibit quantum features. It was named so in relation to the quantum mechanics established by Bohr, Heisenberg, Schrödinger, and many others. The latter deals with only properties and motion of microscopic particles in linear systems, and will be referred to as the linear quantum mechanics (LQM) in this book.

The concept of nonlinearity in quantum mechanics was first proposed by de Broglie in the 1950s in his book, "Nonlinear wave mechanics". LQM had difficulties explaining certain problems right from the start. de Broglie attempted to clarify and solve these problems of LQM using the concept of nonlinearity. Even though a great idea, de Broglie did not succeed because his approach was confined to the framework of the original LQM.

Looking back to the modern history of physics and science, we know that quantum mechanics is really the foundation of modern science. It had great successes in solving many important physical problems, such as the light spectra of hydrogen and hydrogen-like atoms, the Lamb shift in these atoms, and so on. Jargons such as "quantum jump" have their scientific origins and become ever fashionable in our normal life. In this particular case, the phrase "quantum jump" gives a vivid description for major qualitative changes and is almost universally used. However, it was also known that LQM has its problems and difficulties related to the fundamental postulates of the theory, for example, the implications of the uncertainty principle between conjugate dynamical variables, such as position and momentum. Different opinions on how to resolve such issues and further develop quantum mechanics lead to intense arguments and debates which lasted almost a century. The long-time controversy showed that these problems cannot be solved within the framework of LQM. It was also through such debates that the direction to take for improving and further developing quantum mechanics became clear, which was to extend the theory from the linear to the nonlinear regime. Certain fundamental assumptions such as the principle of linear superposition, linearity of the dynamical equation and the independence of the Hamiltonian of a system on its wave function must be abandoned because they are the roots of the problems of LQM. In other words, a new nonlinear quantum theory should be developed.

A series of nonlinear quantum phenomena including the macroscopic quantum effects and motion of solitons or solitary waves have, in recent decades, been discovered one after another from experiments in superconductors, superfluid, ferromagnetic, antiferromagnetic, organic molecular crystals, optical fiber materials and polymer and biological systems, *etc.* These phenomena did underlie nonlinear quantum mechanics because they could not be explained by LQM. Meanwhile, the theories of nonlinear partial differential equations and of solitary wave have been very well established which build the mathematical foundation of nonlinear quantum mechanics. Due to these developments of nonlinear science, a lot of new branches of science, for example, nonlinear vibrational theory, nonlinear Newton mechanics, nonlinear fluid mechanics, nonlinear optics, chaos, synergetics and fractals, have been established or being developed. In such a case, it is necessary to build the nonlinear quantum mechanics described the law of motion of microscopic particles in nonlinear systems.

However, how do we establish such a theory? Experiences in the study of quantum mechanics for several decades tell us that it is impossible to establish such a theory if we followed the direction of de Broglie *et al.* A completely new way of thinking, a new idea and method must be adopted and developed.

According to this idea we will, first of all, study the properties of macroscopic quantum effects, which is a nonlinear quantum effect on macroscopic scale occurred in some matters, for example, superconductors and superfluid. To be more precise, these effects occur in systems with ordered states over a long-range, or, coherent states, or, Bose-like condensed states, which are formed through phase transitions after a spontaneous symmetry breakdown in the systems by means of nonlinear interactions. These results show that the properties of microscopic particles in the macroscopic quantum systems cannot be well represented by LQM. In these systems the microscopic particles are self-localized to become soliton with wave-corpuscle duality. The observed macroscopic quantum effects are just a result produced by soliton motions of the particles in these systems. Therefore, the macroscopic quantum effect is closely related to the nonlinear interaction and to solitary motion of the particles. The close relations among them prompt us to propose and establish the fundamental principles and the theory of NLQM which describes the properties of microscopic particles in the nonlinear systems. We then demonstrate that the NLQM is truely a self-consistent and complete theory. It has so far enjoyed great successes in a wide range of applications in condensed matter, polymers and biological systems. In exploring these applications, we also obtain many important results which are consistent with experimental data. These results confirm the correctness of the NLQM on one hand, and provide further theoretical understanding to many phenomena occurred in these systems on the other hand.

Therefore, we can say that the experimental foundation of the nonlinear quantum mechanics established is the macroscopic quantum effects, and the coherent phenomena. Its theoretical basis is superconducting and superfluidic theories. Its mathematical framework is the theories of nonlinear partial differential equations and of solitary waves. The elementary principles and theory of the NLQM proposed here are established on the basis of results of research on properties of microscopic particles in nonlinear systems and the close relations among the macroscopic quantum effects, nonlinear interactions and soliton motions. The linearity in the LQM is removed and dependence of Hamiltonian of systems on the state wave function of particles is assumed in this theory. Through careful investigations and extensive applications, we demonstrate that this new theory is correct, self-consistent and complete. The new theory solves the problems and difficulties in the LQM.

One of the authors (X. F. Pang) has been studying the NLQM for about 25 years and has published about 100 papers related to this topic. The newly established nonlinear quantum theory has been reported and discussed in many international conferences, for example, International Conference of Nonlinear Physics (ICNP), International Conference of Material Physics (ICMP), Asia Pacific Physics Conference (APPC), International Workshop of Nonlinear Problems in Science and Engineering (IWNPSE), National Quantum Mechanical Conference of China (NQMCC), *etc.*. Pang also published a monograph entitled "The problems for nonlinear quantum theory" in 1985 and a book entitled "The theory of nonlinear quantum mechanics" in 1994 in Chinese. Pang has also lectured in many Universities and Institutes on this subject. Certain materials in this book are based on the above lecture materials and book. It also incorporates many recent results published by Pang and other scientists related to nonlinear Schrödinger equation and nonlinear Klein-Gordon equations.

Finally, we should point out that the NLQM presented here is completely different from the LQM. It is intended for studying properties and motion of microscopic particles in nonlinear systems, in which the microscopic particles become self-localized particles, or solitons, under the nonlinear interaction. Sources of such nonlinear interation can be intrinsic nonlinearity or persistent self-interactions through mechanisms such as self-trapping, self-condensation, self-focusing and self-coherence by means of phase transitions, sudden changes and spontaneous breakdown of symmetry of the systems, and so on. In such cases, the particles have exactly wave-corpuscle duality, and obey simultaneously the classical and quantum laws of motion, *i. e.*, the nature and properties of the microscopic particle are essentially changed from that in LQM. For example, the position and momentum of a particle can be determined to a certain degree. Thus, the linear feature of theory and the principles for independences of the Hamiltonian of the systems on the state-wave function of particle are completely removed. However, this is not to deny the validity of LQM. Rather we believe that it is an approximate theory which is only suitable for systems with linear interactions and the nonlinear interaction is small and can be neglected. In other words, LQM is a special case of the NLQM. This relation between the LQM and the NLQM is similar to that between the relativity and Newtonian mechanics. The NLQM established here is a necessary result of development of quantum mechanics in nonlinear systems.

The establishment of the NLQM can certainly advance and facilitate further developments of natural sciences including physics, biology and astronomy. Meanwhile, it is also useful in understanding the properties and limitations of the LQM, and in solving problems and difficulties encountered by the LQM. Therefore, we hope that by publishing this book on quantum mechanics in the nonlinear systems would add some value to science and would contribute to our understanding of the wonderful nature.

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